

THE
HEAT ENGINE PROBLEM

BY
CHARLES EDWARD LUCKE, M.S.

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY IN THE
FACULTY OF APPLIED SCIENCE, COLUMBIA UNIVERSITY.

NEW YORK

1902

THE
HEAT ENGINE PROBLEM

BY
CHARLES EDWARD LUCKE, M.S.

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY IN THE
FACULTY OF APPLIED SCIENCE, COLUMBIA UNIVERSITY.



NEW YORK

1902

	PAGE.
SOME NEW WORK ON PROPERTIES OF EXPLOSIVE MIXTURES	12
The Apparatus	12
Neutral Gas Generator	15
Constant Pressure Combustion Gas Calorimeter	16
Constant Volume Combustion Pressure Ratio Chamber	17
Constant Pressure Combustion Volume Ratio Apparatus	18
Results Obtained with Apparatus. Flames in Atmosphere of	
Different Air-Gas Mixtures	20
Constant Pressure Combustion Calorimeter	22
Volume-ratios During Constant Pressure Combustion	26
Pressure-ratios for Constant Volume Combustion	27
Conclusion	30

THE HEAT ENGINE PROBLEM.

INTRODUCTION.

It is now a good many years since the first proposition was made for obtaining work by the heat-transforming action of a perfect gas and though each process as it appeared has been more or less completely worked out by those interested in it to show the possibilities of the system and compare it with others yet no investigation of all systems with their mutual relations has ever been made by a general method. This is desirable because no comparisons can be justly drawn otherwise and it is, unfortunately, true that invariably in the past the best conditions of one system have been selected for comparison with some other system working under indifferent conditions. This may not have been done intentionally, in fact it appears that in many cases the comparison seemed perfectly just to the author but the results are almost valueless as bases of generalization for the purpose of reaching clear notions of comparative value.

A commonly used mode of comparison considers different cycles working through the same temperature range whereas equal quantities of heat for each case will result in different temperature ranges and it is pretty clear that comparisons should be made on the basis of some initial conditions one of which is the heat supplied.

A perfect gas will transform heat into work and considering the gas alone without reference to any engine the fraction of the heat that is transformed is dependent on the relation both in sequence and extent of the operations of heating, cooling, expansion and contraction—in short is dependent on the cycle first and the extent of each cyclic phase secondly. It is first required to find out just how much heat energy will be transformed by each cycle and if other things are equal one should be the best for application to engines. But in this comparison we should consider not only which cycle transforms the largest amount of the heat energy supplied to it into work but also through just what range of pressures, volumes and temperatures these cycles must operate to produce the work. This comparison will be purely mathematical and will, when completed, enable us to select the best cycle or best two cycles as the case may be, *i. e.*, that one or those two cycles

that promise the best returns for the labor spent on designing mechanism to execute the cyclic changes.

Having made the mathematical selection of the cycles best adapted to our purpose we are called upon to consider how to heat or cool to cause expansion or contraction with the means at our command and at the rate required. This second part involves all questions of possibility or practicability of doing what seemed mathematically to be desirable.

To place each of the cycles in proper relation each with the other and to show the physical possibility of executing those promising good returns as power generators is the general problem. More particularly the question resolves itself into a search for an effective competitor of the Otto cycle engine which now is the only good heat engine of the perfect gas sort.

As the work progressed beyond the mathematical analytic stage there appeared a cycle which promised good returns for any labor expended on its development but which has been comparatively neglected. The latter part of the work is taken up with a study of physical and engineering problems entering into the execution of this theoretically desirable cycle in engines and includes the determination of many of the physical constants necessary for computation of designs. In this part also there is set down all the difficulties to be encountered and both the solutions obtained and the need of solutions for those questions still open are noted.

RÉSUMÉ OF WORK AND RESULTS.

The work was taken up in detail as follows, and each section brought to a successful conclusion except where otherwise stated :

PART I.

NEW CLASSIFICATION OF CYCLES AND DIAGRAMS OF SAME IN P. V. & $\theta\phi$ COÖRDINATES.

- Cycle I Isometric heating; adiabatic expansion; isopiestic cooling.
- Cycle IA Isometric heating ; adiabatic expansion ; isometric cooling ; isopiestic cooling.
- Cycle IB Isometric heating ; adiabatic expansion ; isothermal cooling ; isopiestic cooling.
- Cycle IC Isometric heating ; adiabatic expansion ; isothermal cooling.
- Cycle II Adiabatic compression ; isometric heating ; adiabatic expansion ; isopiestic cooling.
- Cycle IIA₂ Adiabatic compression ; isometric heating ; adiabatic expansion ; isometric cooling.
- Cycle IIA₁ Adiabatic compression ; isometric heating ; adiabatic expansion ; isometric cooling ; isopiestic cooling.
- Cycle IIB Adiabatic compression ; isometric heating ; adiabatic expansion ; isothermal cooling ; isopiestic cooling.
- Cycle IIC Adiabatic compression ; isometric heating ; adiabatic expansion ; isothermal cooling.
- Cycle III Adiabatic compression ; isopiestic heating ; adiabatic expansion ; isopiestic cooling.
- Cycle IIIA Adiabatic compression ; isopiestic heating ; adiabatic expansion ; isometric cooling ; isopiestic cooling.
- Cycle IIIB Adiabatic compression ; isopiestic heating ; adiabatic expansion ; isothermal cooling ; isopiestic cooling.
- Cycle IIIC Adiabatic compression ; isopiestic heating ; adiabatic expansion ; isothermal cooling.
- Cycle IV Adiabatic compression ; isothermal heating ; adiabatic expansion ; isopiestic cooling.
- Cycle IVA Adiabatic compression ; isothermal heating ; adiabatic expansion ; isometric cooling ; isopiestic cooling.
- Cycle IVB Adiabatic compression ; isothermal heating ; adiabatic expansion ; isothermal cooling ; isopiestic cooling.

Cycle IVC Adiabatic compression ; isothermal heating ; adiabatic expansion ; isothermal cooling.

Cycle V Adiabatic compression ; any law of heating ; adiabatic expansion ; isopiestic cooling.

$\left. \begin{array}{l} \text{VA} \\ \text{VB} \\ \text{VC} \end{array} \right\}$ Similar meanings to preceding cases.

Cycle VI Atmospheric heating ; isometric cooling ; isothermal cooling.

Cycle VII Atmospheric heating ; adiabatic expansion ; isopiestic cooling ; adiabatic compression.

Cycle VIII Atmospheric heating ; adiabatic expansion ; isothermal cooling.

Cycle IX Atmospheric heating ; adiabatic expansion ; isometric cooling ; adiabatic compression.

Cycle X Atmospheric heating ; adiabatic expansion ; any law of cooling ; adiabatic compression.

FOR EACH CYCLE A FORMULA IS DERIVED EXPRESSING EACH OF THE FOLLOWING VARIABLES AS A FUNCTION OF THE HEAT SUPPLIED H_1 ,

(p, v, T) for every point of the diagram.

H_2 or the heat discharged as unavailable.

$W = H_1 - H_2$ or the amount of energy transformed into work.

$E = I - \frac{H_2}{H_1}$ or the efficiency, the fraction of energy supplied

that becomes transformed to work.

R_ϕ or entropy range.

$M.E.T. = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$ or mean-effective temperature.

R_v or volume range.

$M.E.P. = \frac{w}{R_v}$ or mean-effective pressure.

R_p or pressure range.

$M.E.V. = \frac{w}{R_p}$ or mean effective volume.

R_t or temperature range.

CYCLES COMPARED.

The formulæ derived are here collected and curves drawn in two coördinates. One coördinate is in every case H , the heat sup-

plied and the other coördinate the variable under consideration. This gives one curve in every set for each cycle and as many sets as there are variables. Some of these are less important than others and the former are omitted and a set presented for each of the following.

Curves of Temperature after Heating.

Curves of Pressure " "

Curves of Volume " "

Curves of Temperature after Expansion.

Curves of Pressure " "

Curves of Volumes " "

Curves of Heat Discharged or Abstracted.

Curves of Efficiency.

Curves of Mean Effective Pressure.

Curves of Mean Effective Volume.

Curves of Mean Effective Temperature.

Comparison and interpretation of curves.

Selection of a cycle to be applied to engines, the selection based on theoretic grounds alone.

PART II.

THE EXECUTION OF THE CYCLES BY MECHANISMS.

All heat to be derived from a fire and may be imparted to the gas in three ways: (*a*) Through walls, (*b*) by introduction of hot body, (*c*) internal combustion.

External heating condemned.

Introduction of hot masses impracticable.

Internal combustion advocated.

Internal heating by coal, oil, gas.

Explosive internal combustion.

Non-explosive internal combustion.

Explosive internal combustion in engines discussed.

Explosive engines pretty well known and now receiving much attention hence this question left for study of less well known types.

Other types of internal combustion engine considered alone and in relation to others.

Two typical classes of these non-explosive engines stand far in front of others from every point of view, the Brayton and the Diesel.

Review of cyclic analysis so far as it refers to the three typical cases of the practicable cycles.

Left for further study Diesel, Otto, Brayton and their variations.

Diesel an imperfect Carnot and from analysis may be neglected in comparison with the Brayton for power generation.

This leaves as the cycle worthy of application but little known and not at all recognized, Brayton's with its variations.

Special problems introduced by the internal-combustion method of heating. (a) What specific heat is to be used in calculating rise of temperature during a chemical change, that of the constituents, that of the products, or something different from both. (b) Volume change due to chemical action. (c) Is the heat of combustion of a mass of fuel constant or does this depend on conditions, and if so determine them.

Heat suppression in combustion as evidenced by the discrepancy between observed $\frac{p_2}{p_1}$ and $\frac{v_2}{v_1}$ and theoretic values of the same for the isometric and isopiestic combustion respectively.

Effective specific heat versus effective calorific power.

A variation of calorific value with the method of combustion would give Otto or Brayton the preference in efficiency as the case might be.

The non-explosive, internal combustion engine has three elements, fuel and air supply, fire-box, expansion parts.

Fuel and air supply require no study as pumps and compressors are well-known machines.

Utilization of hot expanding gases in cylinders and turbines has been done and requires only enough study to reduce to good practice; there is nothing of the impossible.

The combustion phase is where the trouble has been, no entirely successful fire-box has yet been proposed that will meet all requirements though some there are that work very well under specified conditions.

The engines of this class would have great elasticity of action in speed, power and direction of motion, they would be able to pull up to an overload and they can be constructed to burn any fuel.

Coal-burning engines built and proposed, typical cuts.

Oil-burning engines built and proposed, typical cuts.

Gas-burning engines built and proposed, typical cuts.

Details of Construction Compared.—Cylinders, igniters, governors, preheaters, regenerators, fuel feeds, gas burners, oil burners, mixers, proportioners, use of water, position of fire.

All cycles possible with the non-explosive internal combustion engine.

The engine built by George Brayton, its abandonment and eclipse by the Otto machine.

Only non-explosive internal-combustion engine working to-day is that of Diesel.

The Diesel engine in practice working not under the modified Carnot cycle but under the Brayton cycle. It is then rather a modified Brayton.

The cause of failure in other attempts at application of Brayton or modified Brayton cycles invariably traceable to the fire-box or method of combustion, therefore investigation of methods of burning oil and gas necessary.

Methods of gas combustion classified.

Combustion of gases and mixtures requiring an atmosphere and producing a *volume* of flame.

Combustion of explosive mixtures by self propagation: (*a*) at rest and (*b*) in motion.

Requirements of proper method for the combustion of explosive mixtures in motion.

Experiments made in search for means to fulfill the requirements.

New method of combustion of explosive mixtures in motion a close approach to ideal.

The explosive gas fire.

Operation of the internal combustion engine by intermittent combustion in which the mixture leaves after passing the inlet valve to expansion space.

Operation of engines with continuous combustion, the fire burning steadily and the inlet valves controlling the burnt hot gases.

PART III.

LIQUID FUEL COMBUSTION.

Oil combustion, a series of physical actions involving a knowledge of gas combustion and to be studied in the light of that knowledge.

Different oil systems differ, (*a*) in the method of producing the vapor or oil-gas, and (*b*) in the methods of causing a meeting of this vapor with the air.

Oil combustion classified.

Historical review of different classes by studying characteristic systems.

Requirements for enclosed pressure system to be used in the internal-combustion engine.

Report of series of experiments having for their aim the discovery of a suitable system.

The "explosive oil fire," as developed, proves satisfactory and suitable.

Some experiments and notes to test the availability of the "explosive oil fire" for other uses.

PART IV.

PHYSICAL PROPERTIES OF EXPLOSIVE MIXTURES.

Historical sketch reviewing present knowledge of the properties of explosive mixtures.

No data sufficient for computation of many of the quantities needed in the application of explosive mixtures to engineering work.

Object of this part not only to discover if possible a properly simple and accurate means for obtaining such data, but also to use the apparatus in the making of such observations as time might permit.

Apparatus designed and used for —

1° Measuring $\left\{ \begin{array}{l} \text{gas,} \\ \text{air,} \\ \text{neutral products of combustion.} \end{array} \right.$

2° Mixing, compressing and storing the mixture.

3° Producing products of combustion by method available for collection and storage.

4° Measuring the heat of combustion of explosive mixtures directly by burning at constant volume.

5° The same as 4° but by burning at constant pressure.

6° Measuring pressures due to constant volume combustion directly.

7° Measuring volume's increases due to constant pressure combustion directly.

PART V.

CONCLUSION.

Review of work done, results attained, and statement of what remains to be done.

A METHOD OF CYCLIC ANALYSIS OF HEAT ENGINES.

HEAT ENGINE CYCLES ANALYZED.

Prime movers are useful when they produce motion in required directions against resistances. Nearly all our machines which in general constitute the resistance to prime movers are designed to be operated through an applied forceful rotary motion; therefore the prime movers that are to be of most service to us in our ordinary working operations must develop forced rotary motion. By far the largest number of these rotary motion prime movers come under the head of Heat Engines. These heat engines may be divided into two classes:

(*a*) Those that do work by utilizing the expansion of a substance when changing from the liquid to the gaseous state.

(*b*) Those that do work by utilizing the expansion of a perfect gas, this expansion being caused in some mysterious way by absorption of heat.

The engines of class *a* usually consist of two parts, a part for the production of the vapor, and a part for the utilization of this vapor, converting an increase of volume into a forced rotary motion, in ordinary language, of a boiler and an engine proper. The amount of work that can be done with a given amount of heat by a prime mover of this class, is definitely known within certain limits, when we know how much liquid can be converted into vapor by this heat, and the relative specific volumes of the liquid and resulting vapor. It therefore depends chiefly on the liquid chosen, and, of course, on the mechanical efficiency of the system as a convertor, or, as we may say, on the design of the machine.

The cycle of operations is: (I.) Add heat to liquid and produce vapor. (II.) Allow vapor to expand to as low a pressure as possible, and then discharge it either as vapor or as a reconverted liquid.

This cycle is unchangeable except in incidental details. On the contrary, however, when we employ a perfect gas to which to apply our heat, and whose expansion gives us our work, we may have a large range of different cycles or series of operations that

may be performed on or by the gas in question. The amount of work done by our expanding gas due to the initial application of a given amount of heat will depend on the manner of heating, method of expansion, ultimate disposition of the gas, and, of course, on the mechanical efficiency of the machine for performing the operations desired, and will depend not at all on the gas chosen. In short, the varying amounts of work that may be done will depend solely on the cycle itself. It is therefore evident that there is considerable importance in knowing just how the cycle can effect this change of ultimate useful work for given heat supplied.

In the actual application of any cyclic principles we find various other questions beside the ultimate useful work that demand attention and study. For example, one cycle requires a larger volume of gas to do same work as another; a larger engine is therefore necessary; some cycles operate under higher temperatures than others; some through wider ranges of temperature and pressure. Many other questions might be cited, but enough are given to show that it is necessary that we study the cycles as such, and obtain a statement of every question in terms of the cycle, before we begin the consideration of the mechanical difficulties involved in its carrying out.

It is possible to cause a similar mass of perfect gas to pass through each of the cycles, and obtain an equation for every variable entering into the cycle in terms of the initial conditions and the quantity of heat supplied. For example, we can write

$$\begin{array}{ll} \text{For cycle I. Efficiency} = E = f_I(H_1 C') \\ \text{For cycle II.} & E = f_{II}(H_1 C'') \\ \text{For cycle III.} & E = f_{III}(H_1 C''') \\ \text{For cycle } n, & E = f_n(H_1 C^n) \end{array}$$

where H_1 is heat supplied, and C a constant.

We thus get a series of curves of efficiency, one for each cycle, in terms of the same variable, and get exact relations of the cycles regarding efficiency at a glance. Instead of efficiency we might have chosen the final volumes or the maximum temperatures.

It is the object of this part to consider the various cycles as above outlined and cause one pound of air to pass through each of the cycles under ideal conditions, and to determine every cyclic variable in terms of H_1 , and arbitrary initial conditions. To pass from ideal conditions to practical ones we need only apply a correction factor.

In what follows we shall not consider how the heat is applied or abstracted, the mechanisms involved, nor the practicability of the process.

The following cycles will be considered :

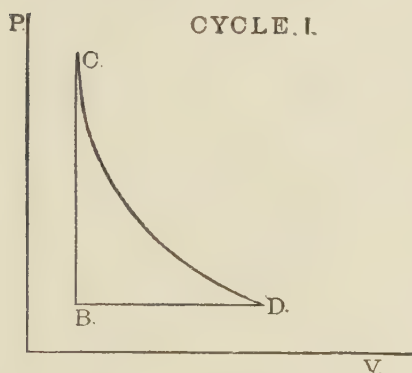


FIG. 1.

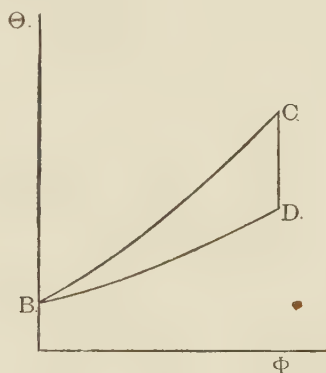


FIG. 2.

Let Fig. 1 be a P.V. and Fig. 2 be a $\Theta\phi$ diagram for the cycle. Then we have :

From *B* to *C*. Addition of heat isometrically from atmospheric pressure.

From *C* to *D*. Adiabatic expansion to atmospheric pressure.

From *D* to *B*. Cooling at atmospheric pressure.

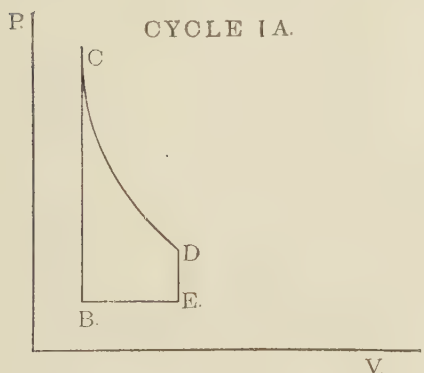


FIG. 3.

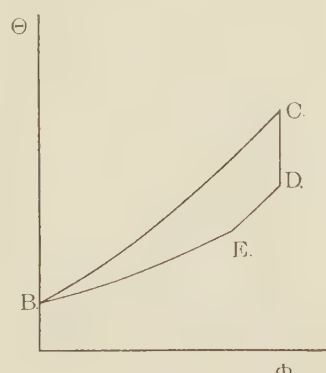


FIG. 4.

We have :

From *B* to *C*. Addition of heat isometrically from atmospheric pressure.

From *C* and *D*. Adiabatic expansion to point above atmospheric pressure.

From D and E . Cooling isometrically to atmospheric pressure.
 From E to B . Cooling at atmospheric pressure.

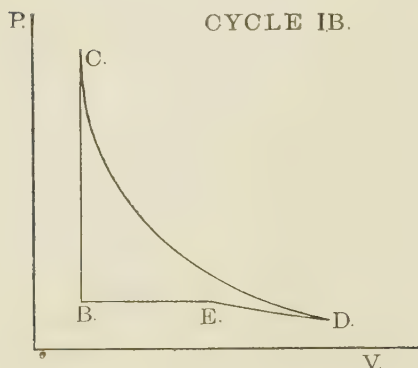


FIG. 5.

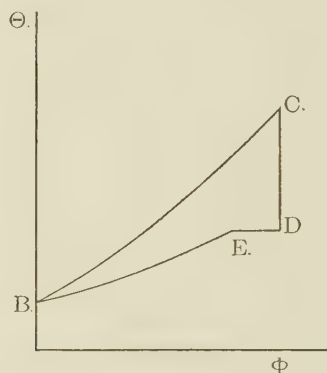


FIG. 6.

We have :

From B to C . Addition of heat isometrically from atmospheric pressure.

From C to D . Adiabatic expansion to below atmospheric pressure.

From D to E . Cooling isothermally to atmospheric pressure.

From E to B . Cooling at atmospheric pressure.

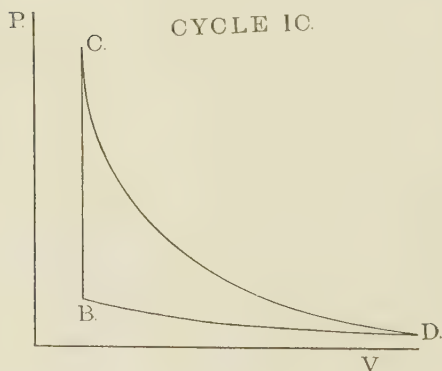


FIG. 7.

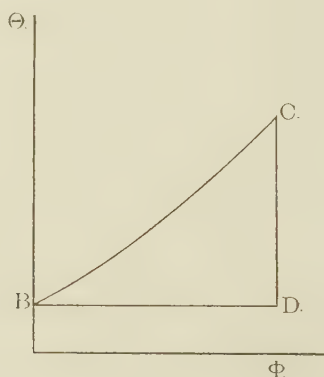


FIG. 8.

We have :

From B to C . Addition of heat isothermally from atmospheric pressure.

From C to D . Adiabatic expansion to pressure below atmosphere such that we get,

From D to B . Cooling isothermally to original volume and atmospheric pressure.

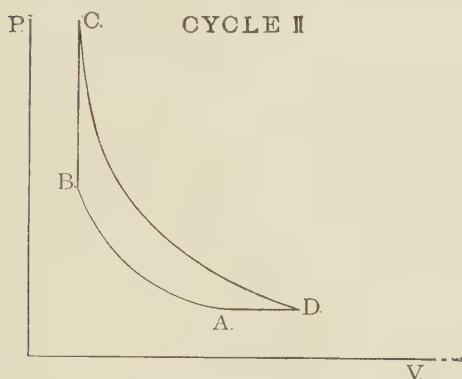


FIG. 9.

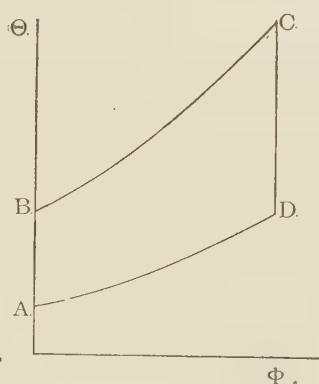


FIG. 10.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isometrically.

From C to D . Adiabatic expansion to atmospheric pressure.

From D to A . Cooling at atmospheric pressure.

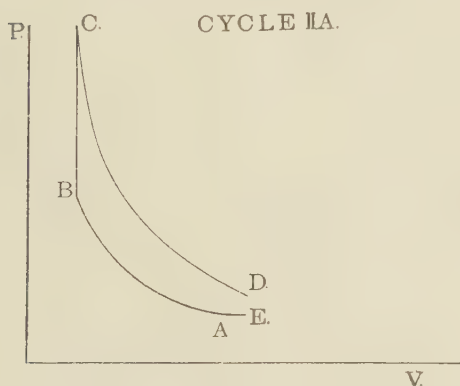


FIG. 11.

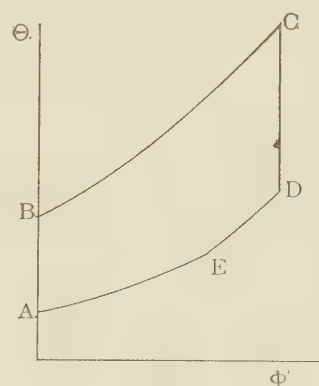


FIG. 12.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isometrically.

From C to D . Adiabatic expansion to pressure above atmosphere.

From D to E . Cooling isometrically to atmosphere.

From E to A . Cooling at atmospheric pressure.

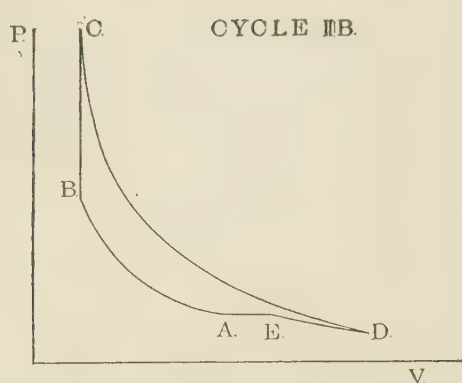


FIG. 13.

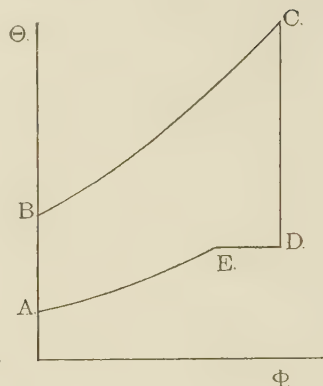


FIG. 14.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isometrically.

From *C* to *D*. Adiabatic expansion to pressure below atmosphere.

From *D* to *E*. Cooling isothermally to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

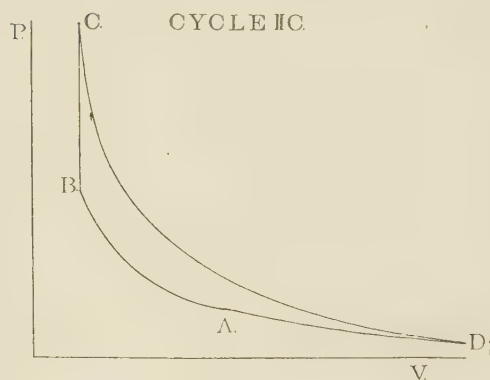


FIG. 15.

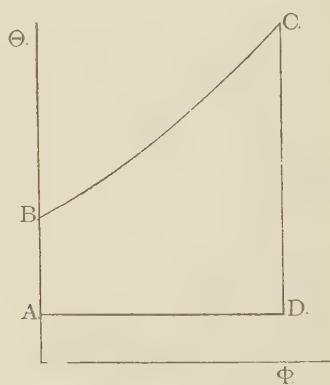


FIG. 16.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isometrically.

From *C* to *D*. Adiabatic expansion to pressure below atmosphere such that we get,

From *D* to *A*. Cooling isothermally to original volume and atmospheric pressure.

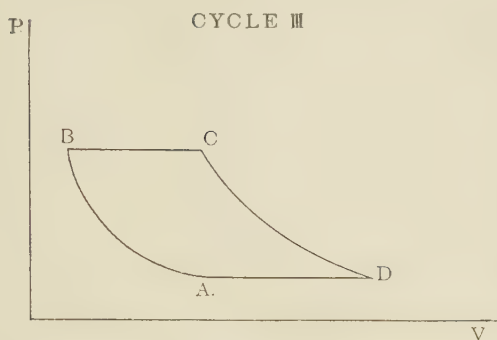


FIG. 17.

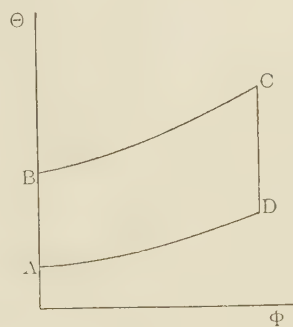


FIG. 18.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to atmospheric pressure.

From D to A . Cooling at atmospheric pressure.

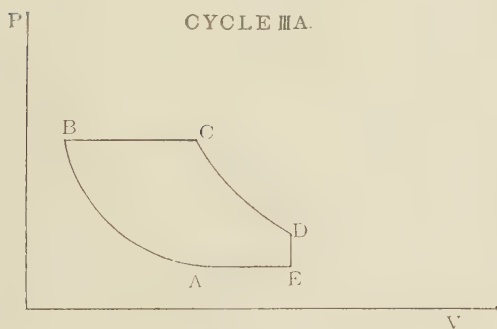


FIG. 19.

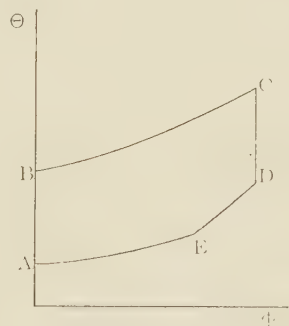


FIG. 20.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to pressure above atmosphere.

From D to E . Cooling isometrically to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

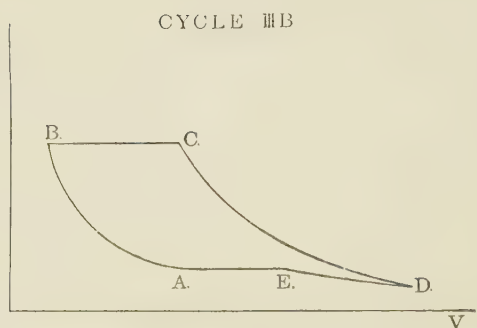


FIG. 21.

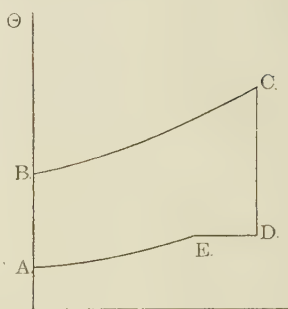


FIG. 22.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isopiastically.

From *C* to *D*. Adiabatic expansion to pressure below atmosphere.

From *D* to *E*. Cooling isothermally to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

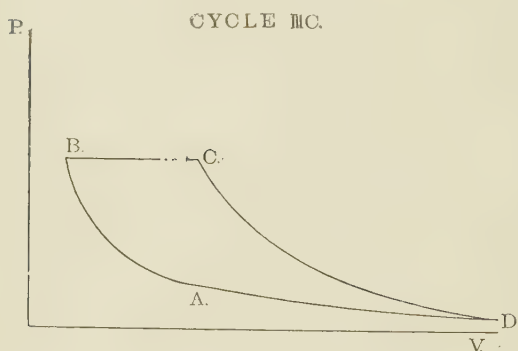


FIG. 23.

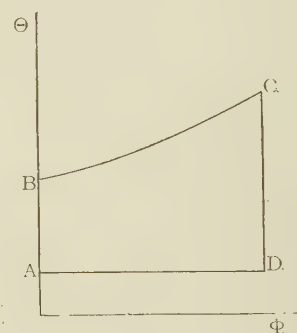


FIG. 24.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isopiastically.

From *C* to *D*. Adiabatic expansion to pressure below atmosphere such that we get,

From *D* to *A*. Cooling isothermally to original volume and atmospheric pressure.

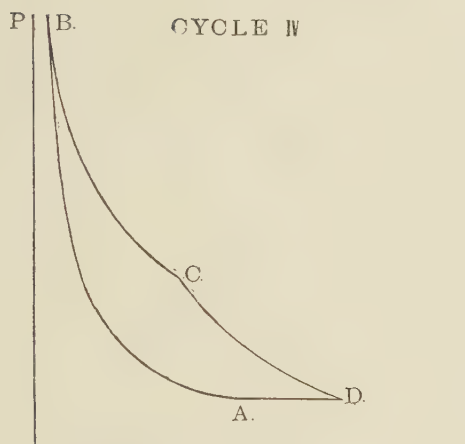


FIG. 25.

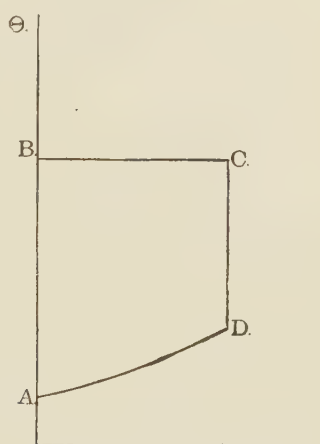


FIG. 26.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isothermally.

From *C* to *D*. Adiabatic expansion to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

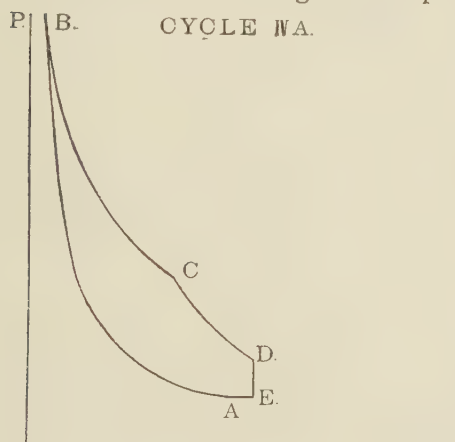


FIG. 27.

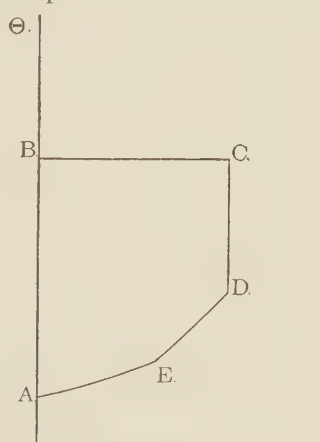


FIG. 28.

We have :

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isothermally.

From *C* to *D*. Adiabatic expansion to pressure above atmosphere.

From *D* to *E*. Cooling isometrically to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

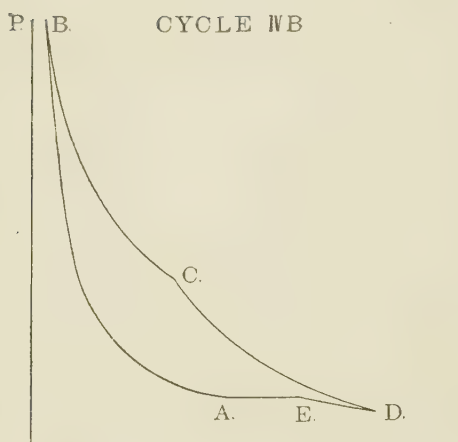


FIG. 29.

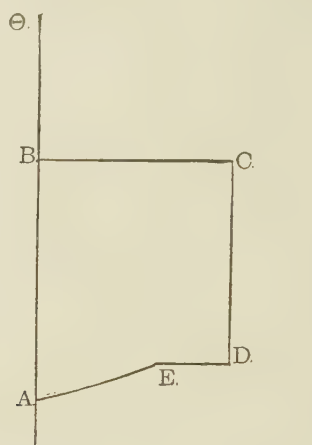


FIG. 30.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isothermally.

From C to D . Adiabatic expansion to pressure below atmosphere.

From D to E . Cooling isothermally to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

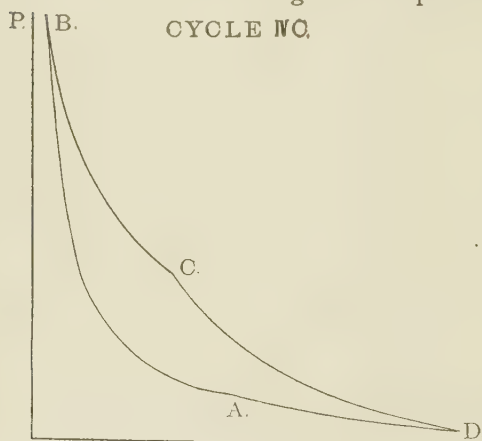


FIG. 31.

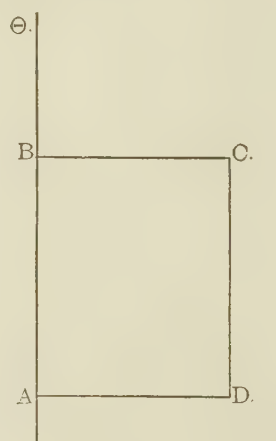


FIG. 32.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isothermally.

From C to D . Adiabatic expansion to pressure below atmosphere such that we get,

From D to A . Cooling isothermally to original volume and atmospheric pressure.

CYCLE V.

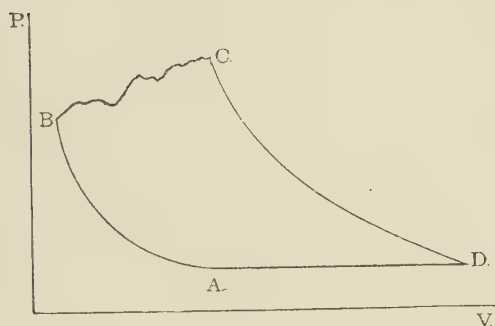


FIG. 33.

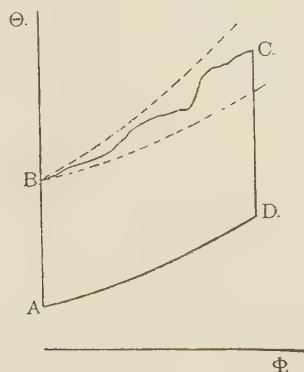


FIG. 34.

We have :

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat at variable pvd .

From C to D . Adiabatic expansion to atmospheric pressure.

From D to A . Cooling at atmospheric pressure.

Cycles V., A , B and C may have the same modification on Cycle V. as II. A , B and C have on III., for example.

THE ATMOSPHERIC OR VACUUM CYCLES.

Here all the cyclic operations take place at or below atmospheric pressure.

CYCLE VI.

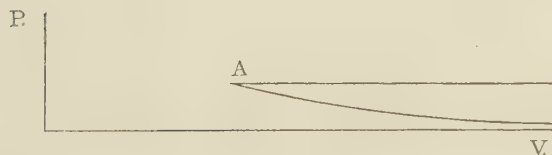


FIG. 35.

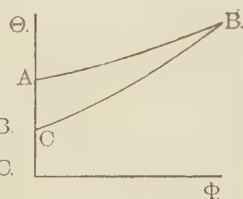


FIG. 36.

We have :

From A to B . Addition of heat at atmospheric pressure.

From B to C . Cooling isometrically.

From C to A . Adiabatic compression.

CYCLE VII.

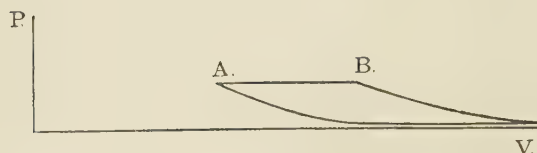


FIG. 37.

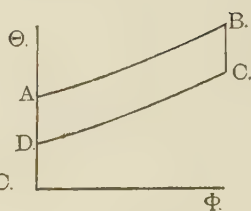


FIG. 38.

We have :

From *A* to *B*. Addition of heat at atmospheric pressure.

From *B* to *C*. Adiabatic expansion.

From *C* to *D*. Cooling isopiesticly.

From *D* to *A*. Adiabatic compression.

CYCLE VIII.

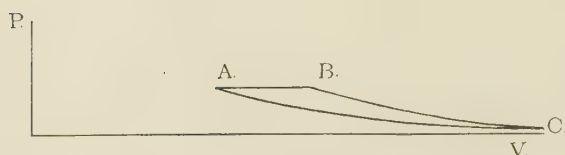


FIG. 39.

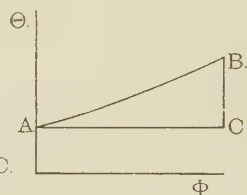


FIG. 40.

We have :

From *A* to *B*. Addition of heat at atmospheric pressure.

From *B* to *C*. Adiabatic compression to such a pressure, that we get,

From *C* to *D*. Isothermal compression to original state.

CYCLE IX.

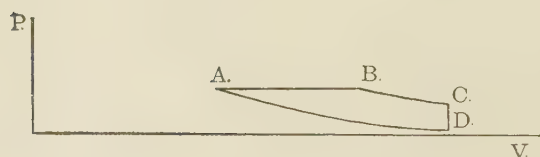


FIG. 41.

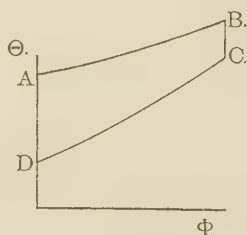


FIG. 42.

We have :

From *A* to *B*. Addition of heat at atmospheric pressure.

From *B* to *C*. Adiabatic expansion.

From *C* to *D*. Cooling isometrically.

From *D* to *A*. Compression adiabatically.

We might have many modifications of these but as a discussion of the type throws sufficient light on the variations considering the importance of the cycles, these modifications will not be discussed.

CYCLE I.

Fig. 1.

Fig. 2.

Let H_1 be the heat added from B to C .

Let C_v be the specific heat of gas at constant volume, and here assumed constant for simplification. It is probably a variable, but so assuming it gives unmanageable formulæ. A correction may afterward be applied, if desired. C_v = heat to raise one pound gas 1° F. at constant volume.

Let v_b be the volume of the gases at point B of the diagram, *i. e.*, before heating and expressed in cubic feet.

Let p_b be the corresponding pressure in pounds per square foot.

Let T_b be the corresponding temperature in absolute degrees Fahrenheit.

Then will the increase in temperature be given by

$$T_c - T_b = \frac{H_1}{C_v}$$

or

$$T_c = T_b + \frac{H_1}{C_v}. \quad (1)$$

Since volume is constant from B to C ,

$$\frac{p_c}{p_b} = \frac{T_c}{T_b}$$

whence

$$p_c = p_b \frac{T_c}{T_b}$$

From (1)

$$\frac{T_c}{T_b} = 1 + \frac{H_1}{C_v T_b}$$

Since this quantity

$$1 + \frac{H_1}{C_v T_b}$$

will enter into many of our equations, let us denote it by

$$1 + \frac{H_1}{C_v T_b} = X$$

whence

$$p_c = p_b X. \quad (2)$$

The adiabatic relation

$$p_d v_d^\gamma = p_c v_c^\gamma$$

gives

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^\frac{1}{\gamma}.$$

But $p_d = p_b$ by hypothesis, hence

$$v_d = v_c \left(\frac{p_c}{p_b} \right)^\frac{1}{\gamma} = v_b (X)^\frac{1}{\gamma}. \quad (3)$$

Another adiabatic relation gives

$$\frac{T_d}{T_c} = \left(\frac{p_d}{p_c} \right)^\frac{\gamma-1}{\gamma}$$

whence

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^\frac{\gamma-1}{\gamma}$$

remembering $p_d = p_b$ and substituting the value of T_c

$$T_d = T_b (X) \left(\frac{1}{X} \right)^\frac{\gamma-1}{\gamma} = T_b X^\frac{1-\gamma+\gamma}{\gamma} = T_b X^\frac{1}{\gamma}. \quad (4)$$

Let H_2 be the heat discharged. Then

$$H_2 = C_p (T_d - T_b).$$

Where C_p = specific heat at constant pressure and assumed constant. Hence substituting

$$H_2 = C_p \left(T_b \left[1 + \frac{H_1}{C_v T_b} \right]^\frac{1}{\gamma} - T_b \right) = C_p T_b (X^\frac{1}{\gamma} - 1). \quad (5)$$

The work done in heat units will be

$$W = H_1 - H_2 \quad (6)$$

$$= H_1 - C_p T_b (X^\frac{1}{\gamma} - 1). \quad (7)$$

And in foot pounds

$$W = J [H_1 - C_p T_b (X^\frac{1}{\gamma} - 1)].$$

This work of expansion could have been obtained by temperatures and by integration as well.

We have

$$W = C_v (T_c - T_b) - C_p (T_d - T_b).$$

But

$$C_p = C_v + \frac{p_0 v_0}{T_0} \times \frac{1}{J}$$

$$\therefore W = C_v T_c - C_v T_b - C_v T_d + C_v T_b - \frac{1}{J} \left[\left(\frac{p_0 v_0}{T_0} \right) T_d + \left(\frac{p_0 v_0}{T_0} \right) T_b \right].$$

We know also

$$\frac{p_0 v_0}{T_0} T_d = \frac{p_d v_d}{T_d} T_d = p_d v_d,$$

$$\frac{p_0 v_0}{T_0} T_b = p_b v_b$$

and

$$p_b = p_d.$$

$$\therefore W = C_v (T_c - T_d) - \frac{1}{J} p_b (v_d - v_b)$$

in heat units. This second term is the area of the rectangle between $\begin{cases} p = 0 \\ p = \text{atmosphere} \end{cases}$ and $\begin{cases} v = v_b \\ v = v_d \end{cases}$ and lying below atmosphere is not available for work.

By integration $W' = \int_{v_d}^{v_c} p dv = \text{area between expansion curve and axis of volumes.}$ The expansion is adiabatic.

$$\therefore W' = p_1 v_1^\gamma \int_{v_d}^{v_c} \frac{dv}{v^\gamma} = - \frac{p_1 v_1^\gamma}{\gamma - 1} \left[\frac{1}{v^\gamma - 1} \right]_{v_d}^{v_c}$$

$$\therefore W' = \frac{p_d v_d^\gamma v_d^{1-\gamma}}{1 - \gamma} - \frac{p_c v_c^\gamma v_c^{1-\gamma}}{1 - \gamma}.$$

$$\therefore W' = \frac{p_d v_d - p_c v_c}{1 - \gamma}.$$

Since

$$1 - \gamma = \frac{C_v - C_p}{C_v}$$

$$\begin{aligned} W' &= \frac{C_v - C_p}{C_v} \left(\frac{p_d v_d}{T_d} \right) - \frac{C_v - C_p}{C_v} \left(\frac{p_c v_c}{T_c} \right) = J C_v (T_c - T_d) \\ &= J C_v (T_c - T_d) \text{ in foot pounds.} \end{aligned}$$

Subtracting the rectangle $p_b (v_d - v_b)$ we get

$$W_1 = J C_v (T_c - T_d) - p_b (v_d - v_b)$$

in foot pounds, or in heat units

$$W = H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)$$

as before.

Before going farther let us apply a test to each of the states B , C , D from the law of perfect gases

$$\begin{aligned}\frac{p_b v_b}{T_b} &= R \\ \frac{p_c v_c}{T_c} &= \frac{p_b X v_b}{T_b X} = \frac{p_b v_b}{T_b} = R \\ \frac{p_d v_d}{T_d} &= \frac{p_b v_b (X)^{\frac{1}{\gamma}}}{T_b (X)^{\frac{1}{\gamma}}} = \frac{p_b v_b}{T_b} = R\end{aligned}$$

hence these are identities as they should be.

Denote the volume swept through or volume range by R_v . Then will

$$R_v = v_d - v_b = v_d - v_c = v_b [X^{\frac{1}{\gamma}} - 1] \quad (8)$$

Whence mean effective pressure

$$\text{M.E.P.} = \frac{W}{R_v} = J \frac{\{H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)\}}{v_b [X^{\frac{1}{\gamma}} - 1]} \quad (9)$$

Efficiency

$$E = \frac{W}{H_1} = 1 - \frac{H_2}{H_1} = 1 - \frac{C_p T_b (X^{\frac{1}{\gamma}} - 1)}{H_1} \quad (10)$$

The entropy range is given by

$$\varphi_c - \varphi_b = C_v \log_e \frac{T_c}{T_b} = C_v \log_e X. \quad (11)$$

Mean effective temperature

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + C_p T_b (X^{\frac{1}{\gamma}} - 1)}{C_v \log_e X} \right). \quad (12)$$

The temperature range

$$R_T = T_c - T_b = \frac{H_1}{C_v}. \quad (13)$$

The pressure range

$$R_p = p_c - p_b = p_b (X - 1). \quad (14)$$

Whence we may write a mean effective volume

$$\text{M.E.V.} = \frac{W}{R_p} = J \frac{\{H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)\}}{p_b (X - 1)} \quad (15)$$

$$R_T = T_c - T_b = T_b (X - 1). \quad (16)$$

These results are here tabulated for reference and comparison with what follows:

We might take the formulæ* derived for mean effective temperature, but as these were the results of a comparison of cycles, none of which ran below atmospheric pressure, it would be better to take another standard here. Let us take arbitrarily as the mean effective temperature one half the sum of the mean temperature of heat addition and the mean temperature of heat abstraction.

$$\text{CYCLE I.} \quad X = 1 + \frac{H_1}{C_c T_b}$$

	Symbol.	Formula as Derived.	Formula Reduced to Initial Conditions.
B	p_b	Arbitrary	p_b
	v_b	"	v_b
	T_b	$\frac{p_b v_b}{R}$	$\frac{p_b v_b}{R}$
C	p_c	$p_b \frac{T_c}{T_b}$	$p_b X$
	v_c	v_b	v_b
	T_c	$T_b + \frac{H_1}{C_v}$	$T_b X$
D	p_d	p_b	p_b
	v_d	$v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}$	$v_c X^{\frac{1}{\gamma}}$
	T_d	$T_c \left(\frac{p_d}{p_c} \right)^{\frac{1}{\gamma}}$	$T_b X^{\frac{1}{\gamma}}$
	H_2	$C_p (T_d - T_b)$	$C_p T_b (X^{\frac{1}{\gamma}} - 1)$
	W	$J(H_1 - H_2)$	$J\{H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)\}$

* *School of Mines Quarterly*, XXI., 4, 1900.

Symbol.	Formula as Derived.	Formula Reduced to Initial Conditions.
E	$\frac{W}{H_1}$	$1 - \frac{C_p T_b (X^{\frac{1}{\gamma}} - 1)}{H_1}$
R_v	$v_d - v_c$	$v_b (X^{\frac{1}{\gamma}} - 1)$
M.E.P.....	$J \frac{W}{R_v}$	$J \left(\frac{H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)}{v_b (X^{\frac{1}{\gamma}} - 1)} \right)$
R_p	$p_c - p_b$	$p_b (X - 1)$
M.E.V.....	$\frac{W}{R_p}$	$J \left(\frac{H_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)}{p_b (X - 1)} \right)$
R_ϕ	$C_v \log_e \frac{T_c}{T_b}$	$C_v \log_e X$
M.E.T.....	$\frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$	$\frac{1}{2} \left(\frac{H_1 + C_p T_b (X^{\frac{1}{\gamma}} - 1)}{C_v \log_e X} \right)$
R_T	$T_c - T_b$	$T_b (X - 1)$

CYCLE I. A.

FIG. 3.

FIG. 4.

We have as in Cycle I. for point C.

$$v_c = v_b \quad (1)$$

$$p_c = p_b X \quad (2)$$

$$T_c = T_b X. \quad (3)$$

Assume

$$p_c > p_d > p_b.$$

Then from the adiabatic relation

$$p_d = p_c \left(\frac{v_c}{v_d} \right)^\gamma$$

or

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}. \quad (4)$$

Also

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}}. \quad (5)$$

Substituting, values of p_c and T_c in (4) and (5) we get

$$v_d = v_b \left(\frac{p_b X}{p_d} \right)^{\frac{1}{\gamma}} = v_b X^{\frac{1}{\gamma}} \left(\frac{p_b}{p_d} \right)^{\frac{1}{\gamma}} \quad (6)$$

$$T_d = T_b X \left(\frac{p_d}{p_b X} \right)^{\frac{\gamma-1}{\gamma}} = T_b X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_b} \right)^{\frac{\gamma-1}{\gamma}}. \quad (7)$$

If we write

$$\frac{p_b}{p_d} = n,$$

then

$$v_d = v_b (Xn)^{\frac{1}{\gamma}} \quad (8)$$

$$T_d = T_b X^{\frac{1}{\gamma}} n^{\frac{1}{\gamma}-1} = \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n}, \quad (9)$$

$$\begin{aligned} v_e &= v_d = v_b (Xn)^{\frac{1}{\gamma}}, \\ p_e &= p_b, \end{aligned} \quad (10)$$

$$T_e = T_b \frac{v_d}{v_b} = T_b (Xn)^{\frac{1}{\gamma}}. \quad (11)$$

Let us apply the perfect gas law to the points B , C , D and E .

$$\frac{p_b v_b}{T_b} = R,$$

$$\frac{p_c v_c}{T_c} = \frac{p_b X v_b}{T_b X} = R,$$

$$\frac{p_d v_d}{T_d} = \frac{p_b v_b (Xn)^{\frac{1}{\gamma}}}{\frac{1}{T_b X^{\frac{1}{\gamma}} n} (Xn)^{\frac{1}{\gamma}}} = R,$$

$$\frac{p_e v_e}{T_e} = \frac{p_b v_b (Xn)^{\frac{1}{\gamma}}}{T_b (Xn)^{\frac{1}{\gamma}}} = R.$$

Heat is abstracted in two parts, the first at constant volume from D to E and the second at constant atmospheric pressure from E to B .

Hence

$$\begin{aligned}
 H_2 &= C_v(T_d - T_e) + C_p(T_e - T_b) \\
 &= C_v \left[T_b \frac{1}{n} (Xn)^{\frac{1}{\gamma}} - T_b (Xn)^{\frac{1}{\gamma}} \right] + C_p \left[T_b (Xn)^{\frac{1}{\gamma}} - T_b \right] \\
 &= C_v T_b (Xn)^{\frac{1}{\gamma}} \left[\frac{1}{n} - 1 \right] + C_p T_b \left[(Xn)^{\frac{1}{\gamma}} - 1 \right]. \quad (12)
 \end{aligned}$$

The work done in foot pounds is

$$W = J(H_1 - H_2) \quad (13)$$

$$\therefore W = J \left(H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) + C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1] \right).$$

$$\therefore E = 1 - \frac{H_2}{H_1} \quad (14)$$

$$\therefore E = 1 - \frac{C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) + C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{H_1} \quad (15)$$

The mean effective pressure

$$\text{M.E.P.} = \frac{W}{R_v} \quad (16)$$

But

$$R_v = v_d - v_b = v_b [(Xn)^{\frac{1}{\gamma}} - 1]. \quad (17)$$

$$\therefore \text{M.E.P.} = J \left\{ \frac{H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) + C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{v_b [(Xn)^{\frac{1}{\gamma}} - 1]} \right\} \quad (18)$$

$$R_p = p_c - p_b = p_b (X - 1) \quad (19)$$

$$\text{M.E.V.} = \frac{W}{R_p}$$

$$= J \left\{ \frac{H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) + C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{p_b (X - 1)} \right\} \quad (20)$$

As before, the entropy range is

$$R_\phi = C_v \log_e X. \quad (21)$$

Taking the mean effective temperature as the mean of the average heating temperature and the average cooling temperature,

$$\text{M.E.T.} = \frac{T' + T''}{2} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) \quad (22)$$

$$= \frac{1}{2} \left\{ \frac{H_1}{C_v \log_e X} + \frac{C_v T_b (Xn)^\gamma \left(\frac{1}{n} - 1 \right) + C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{C_v \log_e X} \right\}. \quad (23)$$

The temperature range is

$$R_T = T_c - T_b = T_b(X - 1). \quad (24)$$

The pressure range is

$$R_p = p_c - p_b = p_b(X - 1). \quad (25)$$

Whence

$$\begin{aligned} \text{M.E.V.} &= \frac{W}{R_p} \\ &= J \frac{H_1 - C_v T_b X^{\frac{1}{\gamma}} \left[\left(\frac{p_d}{p_b} \right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{p_b}{p_d} \right)^{\frac{1}{\gamma}} \right] - C_p T_b \left[X^{\frac{1}{\gamma}} \left(\frac{p_b}{p_d} \right)^{\frac{1}{\gamma}} - 1 \right]}{p_b(X - 1)} \\ &= J \left\{ \frac{H_1 - C_v T_b (Xn)^\gamma \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{p_b(X - 1)} \right\}. \end{aligned} \quad (26)$$

Tabulating these results we get for

CYCLE I. A.

Symbol. Formula as first derived. Formula reduced to initial condition.

$$\begin{aligned} B \left\{ \begin{array}{l} p_b \dots \text{Arbitrary} \dots p_b \\ v_b \dots \dots \dots v_b \\ T_b \dots \dots \frac{p_b v_b}{R} \dots \dots \frac{p_b v_b}{R} \end{array} \right. \\ \\ C \left\{ \begin{array}{l} p_c \dots \dots p_b \frac{T_c}{T_b} \dots \dots p_b X \\ v_c \dots \dots v_b \dots \dots v_b \\ T_c \dots T_b \left(1 + \frac{H_1}{C_v T_b} \right) \dots \dots T_b X \end{array} \right. \end{aligned}$$

Symbol. Formula as first derived.

Formula reduced to initial condition.

$$D \left\{ \begin{array}{l} p_a \dots p_c > p_a > p_b \dots \dots \dots p_c > p_a > p_b \\ v_a \dots v_c \left(\frac{p_c}{p_a} \right)^{\frac{1}{\gamma}} \dots \dots \dots v_b (Xn)^{\frac{1}{\gamma}} \\ T_a \dots T_c \left(\frac{p_a}{p_c} \right)^{\frac{\gamma-1}{\gamma}} \dots \dots \dots \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \end{array} \right.$$

$$E \left\{ \begin{array}{l} p_e \dots \dots \dots p_b \dots \dots \dots p_b \\ v_e \dots \dots \dots v_a \dots \dots \dots v_b (Xn)^{\frac{1}{\gamma}} \\ T_e \dots \dots \dots T_b \frac{v_e}{v_b} \dots \dots \dots T_b (Xn)^{\frac{1}{\gamma}} \end{array} \right.$$

$$H_2 \dots C_v (T_a - T_c) + C_p (T_e - T_b) = C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]$$

$$W \dots J (H_1 - H_2) = J \left\{ H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1] \right\}$$

$$E \dots \dots \dots 1 - \frac{H_2}{H_1} \dots \dots 1 - \frac{C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{H_1}$$

$$R_v \dots \dots v_a - v_b \dots \dots \dots v_b [(Xn)^{\frac{1}{\gamma}} - 1]$$

$$\text{M.E.P.} \dots \frac{W}{R_v} = J \left\{ \frac{H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{v_b [(Xn)^{\frac{1}{\gamma}} - 1]} \right\}$$

$$R_\phi \dots C_v \log_e \frac{T_c}{T_b} \dots \dots \dots C_v \log_e X$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$$

$$= \frac{1}{2} \left\{ \frac{H_1 + C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{C_v \log_e X} \right\}$$

$$R_p \dots \dots p_c - p_b \dots \dots \dots p_b (X - 1)$$

$$\text{M.E.V.} \dots \frac{W}{R_p} \dots J \left\{ \frac{H_1 - C_v T_b (Xn)^{\frac{1}{\gamma}} \left(\frac{1}{n} - 1 \right) - C_p T_b [(Xn)^{\frac{1}{\gamma}} - 1]}{p_b (X - 1)} \right\}$$

$$R_T \dots \dots T_c - T_b \dots \dots \dots T_b (X - 1).$$

CYCLE I. B.

FIG. 5.

FIG. 6.

As the operations up to the point C , *i. e.*, after addition of heat are the same as in cycle I., we may assume these results :

$$v_c = v_b, \quad (1)$$

$$p_c = p_b X, \quad (2)$$

$$T_e = T_b X. \quad (3)$$

Choose p_d so that

$$p_b > p_d > 0. \quad (4)$$

Expansion CD gives

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}, = v_b X^{\frac{1}{\gamma}} \left(\frac{p_b}{p_d} \right)^{\frac{1}{\gamma}}, \quad v_b (Xn)^{\frac{1}{\gamma}} \text{ if } n = \frac{p_b}{p_d}. \quad (5)$$

Also

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} = T_b X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_b} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n}. \quad (6)$$

From the isothermal relation along DE ,

$$T_e = T_d = \frac{T_c (Xn)^{\frac{1}{\gamma}}}{n} \quad (7)$$

$$p_e = p_b \text{ by hypothesis} \quad (8)$$

$$\therefore v_e = \frac{p_d v_d}{p_b}$$

or

$$v_e = \frac{p_d v_d (Xn)^{\frac{1}{\gamma}}}{p_b} = \frac{v_b (Xn)^{\frac{1}{\gamma}}}{n}. \quad (9)$$

Applying the perfect gas law to the various points

$$\frac{p_b v_b}{T_b} = R$$

$$\frac{p_c v_c}{T_c} = \frac{p_b v_b X}{T_b X} = \frac{p_b v_b}{T_b} = R$$

$$\frac{p_d v_d}{T_d} = \frac{p_d v_b (Xn)^{\frac{1}{\gamma}}}{T_b \frac{p_d}{p_b} (Xn)^{\frac{1}{\gamma}}} = \frac{p_b v_b}{T_b} = R$$

$$\frac{p_e v_e}{T_e} = \frac{p_b v_b \frac{(Xn)^{\frac{1}{\gamma}}}{n}}{T_b \frac{(Xn)^{\frac{1}{\gamma}}}{n}} = \frac{p_b v_b}{T_b} = R.$$

Heat is abstracted in two parts, first a part isothermally and second a part at atmospheric pressure. The part abstracted isothermally is extremely difficult to calculate without the aid of the $\theta\phi$ diagram and its relations.

The entropy range along BC has been found to be

$$R_\phi = \phi_c - \phi_b = C_v \log_e \frac{T_b}{T_e} = C_v \log_e X. \quad (10)$$

Now it is evidently the same so far as entropy range is concerned whether we cool at constant pressure from E to B or heat isopiastically from B to E , thus

$$\phi_e - \phi_b = C_p \log_e \frac{T_e}{T_b}. \quad (11)$$

Hence the entropy range for the isothermal operation will be given by

$$\phi_d - \phi_e = C_v \log_e X - C_p \log_e \frac{T_e}{T_b} \quad (12)$$

$$= C_v \log_e X - C_p \log_e \left[X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_b} \right)^{\frac{\gamma-1}{\gamma}} \right]. \quad (13)$$

This latter isothermal change taking place at temperature $T_e = T_d$ the heat of cooling will be given by

$$T_d \left(C_v \log_e X - C_p \log_e \frac{T_e}{T_b} \right). \quad (14)$$

Hence the total heat abstracted is

$$\begin{aligned} H_2 &= C_p(T_e - T_b) + T_d \left[C_v \log_e X - C_p \log_e \frac{T_e}{T_b} \right] \quad (15) \\ &= C_v \left[\frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} - T_b \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \left[C_v \log_e X - C_p \log_e \frac{(Xn)^{\frac{1}{\gamma}}}{n} \right]. \end{aligned}$$

But

$$\begin{aligned} C_p \log_e \frac{(Xn)^{\frac{1}{\gamma}}}{n} &= C_p \log_e X^{\frac{1}{\gamma}} + C_p \log_e n^{\frac{1}{\gamma}-1} \\ &= C_v \log_e X + (C_v - C_p) \log_e n. \end{aligned}$$

Since

$$\frac{1}{\gamma} = \frac{C_v}{C_p}.$$

And

$$\frac{1}{\gamma} - 1 = \frac{C_v - C_p}{C_p}.$$

Hence

$$H_2 = C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}. \quad (16)$$

The work in foot-pounds is

$$W = J(H_1 - H_2)$$

$$\therefore W = J \left\{ H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] - \frac{T_b X(n)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p} \right\} \quad (17)$$

$$E = 1 - \frac{H_2}{H_1}$$

$$\therefore E = 1 - \frac{C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{H_1} \quad (18)$$

$$R_v = v_e - v_b = v_b \left[(Xn)^{\frac{1}{\gamma}} - 1 \right]. \quad (19)$$

$$\therefore \text{M.E.P.} = J \left\{ \frac{H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b X(n)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{v_b \left[X(n)^{\frac{1}{\gamma}} - 1 \right]} \right\} \quad (20)$$

$$R_p = p_c - p_d = p_b X - p_a \quad (21)$$

$$\therefore \text{M.E.V.} = J \left\{ \frac{H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{p_b X - p_d} \right\} \quad (22)$$

The mean effective temperature being the mean of the heating and cooling means is given by

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$$

where R_ϕ is same as in previous cycle.

$$\therefore \text{M.E.T.} = \frac{1}{2 \log_e X} \left\{ H_1 + C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] \right. \quad (23)$$

$$\left. + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{\frac{1}{c_v - c_p}} \right\}$$

$$R_T = T_b - T_a = T_b \left[1 - \frac{(Xn)^{\frac{1}{\gamma}}}{n} \right]. \quad (24)$$

Tabulating :

CYCLE I. B.

Symbol. Formula as first derived. Formula reduced to initial conditions.

$$B \left\{ \begin{array}{ll} p_b \dots\dots \text{Atmosphere} \dots\dots\dots \text{Atmospheric } p_b \\ v_b \dots\dots \text{Arbitrary} \dots\dots\dots v_b \\ T_b \dots\dots \frac{p_b v_b}{R} \dots\dots\dots \frac{p_b v_b}{R} \end{array} \right.$$

$$C \left\{ \begin{array}{ll} p_c \dots\dots p_b \frac{T_c}{T_b} \dots\dots\dots p_b X \\ v_c \dots\dots\dots v_b \\ T_c \dots\dots T_b \left(1 + \frac{H_1}{C_v T_b} \right) \dots\dots\dots T_b X \end{array} \right.$$

$$D \left\{ \begin{array}{ll} p_d \dots\dots p_b > p_d > 0 \dots\dots\dots p_b > p_d > 0 \\ v_d \dots\dots v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} \dots\dots\dots v_b (Xn)^{\frac{1}{\gamma}} \\ T_d \dots\dots T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} \dots\dots\dots \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \end{array} \right.$$

$$E \left\{ \begin{array}{ll} p_e \dots\dots p_b \dots\dots\dots p_b \\ v_e \dots\dots \frac{p_d v_d}{p_d} \dots\dots\dots \frac{v_b (Xn)^{\frac{1}{\gamma}}}{n} \\ T_e \dots\dots T_d \dots\dots\dots \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \end{array} \right.$$

$$\begin{aligned}
 H_2 \dots\dots\dots C_p(T_e - T_b) + T_d \left[C_v \log_e X - C_p \log_e \frac{T_e}{T_b} \right] \\
 = C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}
 \end{aligned}$$

$$\begin{aligned}
 W \dots\dots\dots J(H_1 - H_2) \\
 = J \left\{ H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] - \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p} \right\}
 \end{aligned}$$

$$\begin{aligned}
 E \dots\dots\dots 1 - \frac{H_2}{H_1} \\
 = 1 - \frac{C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{H_1}
 \end{aligned}$$

$$R_v \dots\dots\dots (v_e - v_b) \dots\dots\dots v_b [(Xn)^{\frac{1}{\gamma}} - 1]$$

$$\begin{aligned}
 \text{M.E.P.} \dots\dots\dots \frac{W}{R_v} \\
 = J \left\{ \frac{H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] - \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{v_b [(Xn)^{\frac{1}{\gamma}} - 1]} \right\}
 \end{aligned}$$

$$R_p \dots\dots\dots p_c - p_d \dots\dots\dots p_b X - p_d$$

$$\begin{aligned}
 \text{M.E.V.} \dots\dots\dots \frac{W}{R_p} \\
 = J \left\{ \frac{H_1 - C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{T_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p}}{p_b X - p_d} \right\}
 \end{aligned}$$

$$R_\phi \dots\dots\dots \log_e \frac{T_e}{T_b} \dots\dots\dots \log_e X$$

$$\begin{aligned}
 \text{M.E.T.} \dots\dots\dots \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) \\
 = \frac{1}{2 \log_e X} \left\{ H_1 + C_p T_b \left[\frac{(Xn)^{\frac{1}{\gamma}}}{n} - 1 \right] + \frac{C_b (Xn)^{\frac{1}{\gamma}}}{n} \log_e n^{c_v - c_p} \right\}
 \end{aligned}$$

$$R_T \dots \dots T_b - T_d \dots \dots T_b \left[1 - \frac{(Xn)^{\frac{1}{\gamma}}}{n} \right]$$

CYCLE I. C.

FIG. 7.

FIG. 8.

Assume all results to point C from Cycle I.

$$p_c = p_b X \quad (1)$$

$$v_c = v_b \quad (2)$$

$$T_c = T_b X. \quad (3)$$

From the adiabatic CD ,

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = v_b \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} \quad (4)$$

This adiabatic must meet the isothermal from B in point D , hence

$$v_d = v_b \frac{p_b}{p_d}. \quad (5)$$

Equate (4) and (5)

$$v_b \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = v_b \frac{p_b}{p_d}$$

$$\therefore p_b^{\frac{1}{\gamma}} = p_d^{\frac{1}{\gamma}} \frac{p_b X}{p_d} p_d^{\gamma-1} p_b X$$

$$\therefore p_d = p_b X^{\frac{1}{1-\gamma}} = \frac{p_b}{X^{\frac{1}{\gamma-1}}}. \quad (6)$$

This is the pressure at which the isothermal through B will meet the adiabatic through C . Its corresponding volume is

$$v_d = v_b \frac{p_b}{p_d X^{\frac{1}{1-\gamma}}} = v_b X^{\frac{1}{\gamma-1}} \quad (7)$$

$$T_d = T_b. \quad (8)$$

The heat abstracted by the isothermal cooling is found as before from $\theta\phi$ relation.

$$\phi_c - \phi_b = C_v \log_e \frac{T_c}{T_b} = C_v \log_e X. \quad (9)$$

Hence

$$\varphi_d - \varphi_b = C_v \log_e X. \quad (10)$$

$$\therefore H_2 = T_b C_v \log_e X. \quad (11)$$

The work done in foot-pounds is

$$W = J(H_1 - H_2) = J(H_1 - T_b C_v \log_e X). \quad (12)$$

The efficiency is

$$E = 1 - \frac{H_2}{H_1} = 1 - \frac{T_b C_v \log_e X}{H_1}. \quad (13)$$

The volume range is

$$R_v = v_d - v_b = v_b (X^{\frac{1}{\gamma-1}} - 1). \quad (14)$$

Hence

$$\text{M.E.P.} = \frac{J(H_1 - T_b C_v \log_e X)}{v_b (X^{\frac{1}{\gamma-1}} - 1)}. \quad (15)$$

The pressure range is

$$R_p = p_c - p_d = p_b X - \frac{p_b}{X^{\gamma-1}} = p_b \left(X - \frac{1}{X^{\gamma-1}} \right). \quad (16)$$

$$\text{M.E.V.} = \frac{J(H_1 - T_b C_v \log_e X)}{p_b \left(X - \frac{1}{X^{\gamma-1}} \right)}. \quad (17)$$

The entropy range was found $R_\phi = C_v \log_e X$ hence

$$\begin{aligned} \text{M.E.T.} &= \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) = \frac{1}{2 C_v \log_e X} (H_1 + T_b C_v \log_e X) \\ &= \frac{1}{2} \left(\frac{H_1}{C_v \log_e X} + T_b \right). \end{aligned}$$

And

$$R_T = T_c - T_b = T_b (X - 1).$$

Tabulate

CYCLE I. C.

Symbol.	Formula as first derived.	Formula reduced to initial conditions.
p_b	Atmospheric.....	Atmospheric p_b
v_b	Arbitrary.....	τ'_b
T_b	$\frac{p_b \tau'_b}{R}$	$\frac{p_b \tau'_b}{R}$

Symbol.	Formula as first derived.	Formula reduced to initial conditions.
$C \left\{ \begin{array}{l} p_c \dots\dots\dots p_b \frac{T_b}{T_c} \dots\dots\dots p_b X \\ v_c \dots\dots\dots v_b \dots\dots\dots v_b \\ T_c \dots\dots\dots T_b \left(1 + \frac{H_1}{C_v T_b} \right) \dots\dots\dots T_b X \end{array} \right.$		
$D \left\{ \begin{array}{l} p_d \dots\dots\dots \left(\frac{p_b}{X^{\frac{1}{\gamma-1}}} \right) \dots\dots\dots \frac{p_b}{X^{\frac{1}{\gamma-1}}} \\ v_d \dots\dots\dots v_b X^{\frac{1}{\gamma-1}} \dots\dots\dots v_b X^{\frac{1}{\gamma-1}} \\ T_d \dots\dots\dots T_b \dots\dots\dots T_b \end{array} \right.$		
$H_2 \dots\dots\dots T_b C_v \log_e \frac{T_c}{T_b} \dots\dots\dots T_b C_v \log_e X$		
$W \dots\dots\dots J(H_1 - H_2) \dots\dots\dots J(H_1 - T_b C_v \log_e X)$		
$E \dots\dots\dots I - \frac{H_2}{H_1} \dots\dots\dots 1 - \frac{T_b C_v \log_e X}{H_1}$		
$R_c \dots\dots\dots v_d - v_b \dots\dots\dots v_b (X^{\frac{1}{\gamma-1}} - 1)$		
$M.E.P. \dots\dots\dots \frac{W}{R_v} \dots\dots\dots J \left\{ \frac{H_1 - T_b C_v \log_e X}{v_b (X^{\frac{1}{\gamma-1}} - 1)} \right\}$		
$R_p \dots\dots\dots p_c - p_d \dots\dots\dots p_b \left(X - \frac{1}{X^{\frac{1}{\gamma-1}}} \right)$		
$M.E.V. \dots\dots\dots \frac{W}{R_p} \dots\dots\dots J \left\{ \frac{H_1 - T_b C_v \log_e X}{p_b \left(X - \frac{1}{X^{\frac{1}{\gamma-1}}} \right)} \right\}$		
$R_\phi \dots\dots\dots C_v \log_e \frac{T_c}{T_b} \dots\dots\dots C_v \log_e X$		
$M.E.T. \dots\dots\dots \frac{1}{2 \log_e X} (H_1 + H_2) \dots\dots\dots \frac{1}{2} \left(\frac{H_1}{C_v \log_e X} + T_b \right)$		
$R_T \dots\dots\dots T_c - T_b \dots\dots\dots T_b (X - 1)$		

CYCLE II.*

Let Fig. 9 be the P. V. and Fig. 10 the $\theta\phi$ diagram of this cycle.

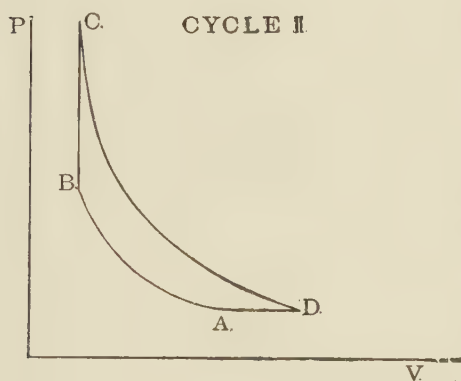


FIG. 9.

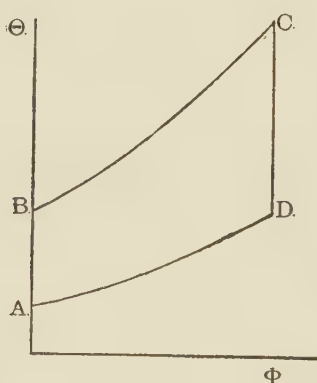


FIG. 10.

In the compression cycles the volume ratio $\frac{v_a}{v_b}$ will enter into many of our formulæ and we will find it convenient to write

$$\frac{v_a}{v_b} = r.$$

The compression is adiabatic, hence

$$p_b = p_a \left(\frac{v_a}{v_b} \right)^\gamma = p_a r^\gamma \quad (1)$$

$$T_b = T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1} = T_a r^{\gamma-1}. \quad (2)$$

During addition of heat $v_c = v_b$ and therefore

$$p_c = p_b \frac{T_c}{T_b}.$$

* *School of Mines Quarterly*, Vol. XXII., April, 1901, No. 3.

If we write $\frac{T_c}{T_b} = 1 + \frac{H_1}{C_c T_b} = X$ as in the previous cycles we get

$$p_c = p_b X = p_a \gamma X \quad (3)$$

$$T_c = T_a \gamma^{\gamma-1} X. \quad (4)$$

Adiabatic expansion gives

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} \text{ or if } p_d = p_a$$

$$v_d = v_b \left(\frac{p_a \gamma X}{p_a} \right)^{\frac{1}{\gamma}} = v_b \gamma X^{\frac{1}{\gamma}} = \frac{v_a}{\gamma} \gamma X^{\frac{1}{\gamma}} = v_a X^{\frac{1}{\gamma}}. \quad (5)$$

$$T_d = T_a \frac{v_d}{v_a} = T_a X^{\frac{1}{\gamma}}. \quad (6)$$

Applying the perfect gas law

$$\frac{p_a v_a}{T_a} = R,$$

$$\frac{p_b v_b}{T_b} = \frac{p_a \gamma v_a}{T_a \gamma^{\gamma-1} \gamma} = R,$$

$$\frac{p_c v_c}{T_c} = \frac{p_a \gamma^{\gamma} X v_a}{T_a \gamma^{\gamma-1} X \gamma} = R,$$

$$\frac{p_d v_d}{T_d} = \frac{p_a v_a X^{\frac{1}{\gamma}}}{T_a X^{\frac{1}{\gamma}}} = R.$$

The heat discharged

$$H_2 = C_p (T_d - T_a) = C_p T_a (X^{\frac{1}{\gamma}} - 1). \quad (7)$$

The work done is

$$W = H_1 - H_2 = H_1 - C_p T_a (X^{\frac{1}{\gamma}} - 1) \quad (8)$$

$$E = \frac{W}{H_1} = 1 - \frac{C_p T_a (X^{\frac{1}{\gamma}} - 1)}{H_1} \quad (9)$$

$$R_v = v_d - v_b = v_b \left(X^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right) \quad (10)$$

$$\text{M.E.P.} = \frac{W}{R_v} = J \frac{H_1 - C_v T_a (X^{\frac{1}{\gamma}} - 1)}{v_a \left(X^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right)} \quad (11)$$

$$R_\phi = C_v \log_e X \quad (12)$$

$$\therefore \text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_v} \right) = \frac{1}{2} \left(\frac{H_1 + C_p T_a (X^{\frac{1}{\gamma}} - 1)}{C_v \log_e X} \right) \quad (13)$$

$$R_p = p_c - p_a = p_a (\gamma^\gamma X - 1) \quad (14)$$

$$\text{M.E.V.} = J \frac{1 - C_v T_a (X^{\frac{1}{\gamma}} - 1)}{p_a (\gamma^\gamma X - 1)} \quad (15)$$

CYCLE II.

Symbol.	Formula as First Derived.	Formula Reduced.
p_b	$p_a \left(\frac{v_a}{v_b} \right)^\gamma$	$p_a \gamma^\gamma$
v_b	$\frac{v_a}{\gamma}; (\gamma \text{ arbitrary})$	$\frac{v_a}{\gamma}$
T_b	$T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
p_c	$p_b \frac{T_c}{T_b}$	$p_a \gamma^\gamma X$
v_c	v_b	$\frac{v_a}{\gamma}$
T_c	$T_b \left(1 + \frac{H_1}{C_p T_b} \right)$	$T_b X$

Symbol.	Formula as First Derived.	Formula Reduced.
p_d	p_a	p_a
v_d	$v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}$	$v_a X^{\frac{1}{\gamma}}$
T_d	$T_a \frac{v_d}{v_a}$	$T_a X^{\frac{1}{\gamma}}$
H_2	$C_p(T_d - T_a)$	$C_p T_a (X^{\frac{1}{\gamma}} - 1)$
W	$H_1 - H_2$	$H_1 - C_p T_a (X^{\frac{1}{\gamma}} - 1)$
E	$1 - \frac{H_2}{H_1}$	$\frac{1 - C_p T_a (X^{\frac{1}{\gamma}} - 1)}{H_1}$
R_v	$v_d - v_b$	$v_a (X^{\frac{1}{\gamma}} - 1)$
M.E.P.....	$J \frac{W}{R_v}$	$J \frac{H_1 - C_p T_a (X^{\frac{1}{\gamma}} - 1)}{v_a (X^{\frac{1}{\gamma}} - 1)}$
R_ϕ	$C_v \log_e \frac{T_c}{T_b}$	$C_v \log_e X$
M.E.T.....	$\frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$	$\frac{1}{2} \left(\frac{H_1 + C_p T_a (X^{\frac{1}{\gamma}} - 1)}{C_v \log_e X} \right)$
R_p	$p_c - p_a$	$p_a (\gamma^{\gamma} X - 1)$
M.E.V.....	$J \frac{W}{R_p}$	$J \frac{H_1 - C_p T_a (X^{\frac{1}{\gamma}} - 1)}{p_a (\gamma^{\gamma} X - 1)}$
R_T	$T_c - T_a$	$T_a (R^{\gamma-1} X - 1)$

CYCLE II. A.

Let Fig. 11 be the P. V. and Fig. 12 the $\theta\phi$ diagram for the cycle.

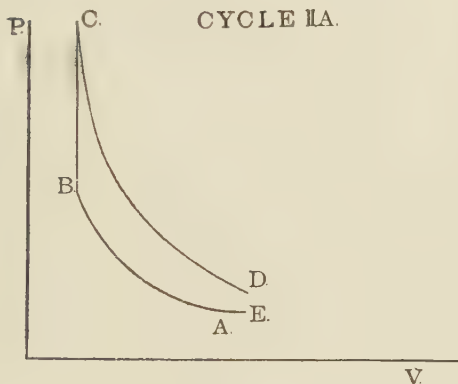


FIG. 11.

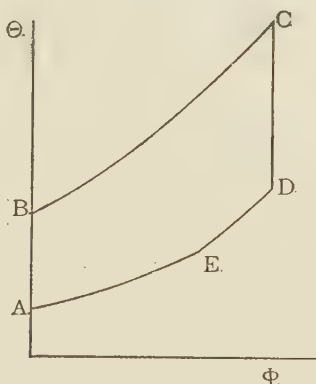


FIG. 12.

Then we have, since the compression is as in Cycle II.,

$$\frac{v'_a}{v_b} = \gamma, \quad (1)$$

$$p_b = p_a \gamma^\gamma, \quad (2)$$

$$T_b = T_a \gamma^{\gamma-1}. \quad (3)$$

Also for C the heat addition being as before

$$v_c = v_b, \quad (4)$$

$$p_c = p_b X = p_a \gamma^\gamma X, \quad (5)$$

$$T_c = T_b X = T_a \gamma^{\gamma-1} X. \quad (6)$$

The point D lies arbitrarily between C and the atmospheric line on the adiabatic

$$p_d = p_c \left(\frac{v_c}{v_d} \right)^\gamma = p_c \left(\frac{v_b}{v_d} \right)^\gamma = p_a \gamma^\gamma X \left(\frac{v'_a}{\gamma v_d} \right)^\gamma = p_a X \left(\frac{v'_a}{v_d} \right)^\gamma. \quad (7)$$

From this point we will consider two cases: 1°, the general where v_d is greater than v_a , and 2°, a particular case where $v_d = v_a$.

$$1^{\circ}. \quad v_d > v_a \text{ and } p_d > p_a$$

$$2^{\circ}. \quad v_d = v_a$$

Then we have

$$p_d = p_a X \left(\frac{v_a}{v_d} \right)^{\gamma} \quad (8)$$

$$p_d' = p_a X \quad (8')$$

$$T_d = T_a \gamma^{\gamma-1} X \left(\frac{v_a}{v_d} \right)^{\gamma-1}$$

$$\therefore T_d = T_a X \left(\frac{v_a}{v_d} \right)^{\gamma-1} \quad (9)$$

$$T_d' = T_a X \quad (9')$$

$$v_e = v_d \quad (10)$$

$$v_e' = v_a \quad (10')$$

$$p_e = p_a \quad (11)$$

$$p_e' = p_a \quad (11')$$

$$T_e = T_d \frac{p_e}{p_d} = T_d \frac{p_a}{p_d}$$

$$\therefore T_e = T_d \frac{v_d}{v_a} \quad (12)$$

$$T_e' = T_a \quad (12')$$

Apply the perfect gas law

$$\frac{p_a v_a}{T_a} = R,$$

$$\frac{p_b v_b}{T_b} = \frac{p_a \gamma^{\gamma} v_a}{T_a \gamma^{\gamma-1} \gamma} = R,$$

$$\frac{p_c v_c}{T_c} = \frac{p_a \gamma^{\gamma} X v_a}{T_a \gamma^{\gamma-1} X \gamma} = R.$$

$$1^{\circ}. \quad \frac{p_d v_d}{T_d} = \frac{p_a X \left(\frac{v_a}{v_d} \right)^{\gamma} v_d}{T_a X \left(\frac{v_a}{v_d} \right)^{\gamma-1}} = R,$$

$$\frac{p_e v_e}{T_e} = \frac{p_a v_d}{T_a \frac{v_d}{v_a}} = R.$$

$$2^{\circ}. \quad \frac{p_d' v_d'}{T_d'} = \frac{p_a X v_a}{T_a X} = R,$$

$$\frac{p_e' v_e'}{T_e'} = \frac{v_a p_a}{T_a} = R.$$

Heat is abstracted as follows :

$$H_2 = C_v(T_d - T_e) + C_p(T_e - T_a) \quad (13) \qquad H_2' = C_v(T_d - T_a) \quad (13')$$

$$= C_v T_a \left[X \left(\frac{v_a}{v_d} \right)^{\gamma-1} - \frac{v_d}{v_a} \right] + C_p T_a \left[\frac{v_d}{v_a} - 1 \right].$$

$$\therefore H_2 = C_v T_a \frac{v_d}{v_a} \left[\left(\frac{v_a}{v_d} \right)^{\gamma} X - 1 \right] + C_p T_a \left[\frac{v_d}{v_a} - 1 \right].$$

The work is given by

$$W = H_1 - H_2 \qquad W' = H_1 - C_v T_a (X - 1) \quad (14')$$

$$= H_1 - C_v T_a \frac{v_d}{v_a} \left[X \left(\frac{v_a}{v_d} \right)^{\gamma} - 1 \right] - C_p T_a \left[\frac{v_d}{v_a} - 1 \right]. \quad (14)$$

Volume range is

$$R_v = v_d - v_b = v_d - \frac{v_a}{\gamma} \quad (15) \qquad R_v' = v_a - v_b = v_a \left(1 - \frac{1}{\gamma} \right) \quad (15')$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{W}{v_d - \frac{v_a}{\gamma}} \quad (16) \qquad \text{M.E.P.} = J \frac{W'}{v_a \left(1 - \frac{1}{\gamma} \right)} \quad (16')$$

Entropy range is the same for both cases,

$$R\phi = C_v \log_e \frac{T_c}{T_b} = C_v \log_e X. \quad (17)$$

Mean of mean temperatures of heat addition and abstraction,

$$\begin{aligned} \text{M.E.T.} &= \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) \\ &= \frac{1}{2} \frac{H_1 + H_2}{C_v \log_e X} \\ &= \frac{1}{2} \left\{ \frac{H_1 + C_v T_a \frac{v_d}{v_a}}{\left[X \left(\frac{v_a}{v_d} \right)^\gamma - 1 \right]} - \frac{C_p T_a \left(\frac{v_d}{v_a} - 1 \right)}{C_v \log_e X} \right\} \quad (18) \end{aligned} \quad \left| \begin{aligned} \text{M.E.T.'} \\ &= \frac{1}{2} \left\{ \frac{H_1 + C_v T_a (X - 1)}{C_v \log_e X} \right\} \quad (18') \end{aligned} \right.$$

Pressure range is same for both cases

$$\therefore R_p = p_c - p_a = p_a (\gamma^\gamma X - 1). \quad (19)$$

Mean effective volume

$$\text{M.E.V.} = J \frac{W}{p_a (\gamma^\gamma X - 1)} \quad (20) \quad \left| \quad \text{M.E.V.} = J \frac{W'}{p_a (\gamma^\gamma X - 1)} \quad (20') \right.$$

Temperature range is also the same for both

$$R_T = T_c - T_a = T_a (\gamma^{\gamma-1} X - 1) \quad (21)$$

CYCLE II. B.

Let Fig. 13 and Fig. 14 be the P.V. and $O\phi$ diagram respectively of the cycle.

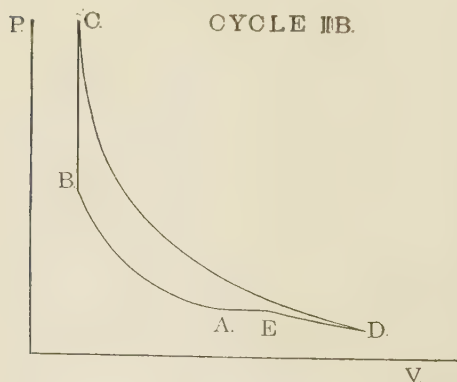


FIG. 13.

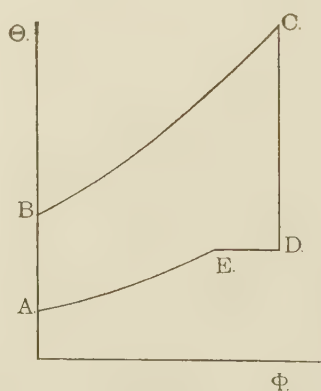


FIG. 14.

Assume same results as before up to the point c . Take p_d something less than atmosphere, *i. e.*,

$$p_a > p_d > 0. \quad (1)$$

Then

$$\tau_d = \tau_c \left(\frac{p_c}{p_d} \right)^\gamma = \frac{\tau_a}{\gamma} \left(\frac{p_a \gamma^\gamma X}{p_d} \right)^\gamma = \tau_a \left(\frac{p_a X}{p_d} \right)^\gamma. \quad (2)$$

And

$$\begin{aligned} T_d &= T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} = T_a \gamma^{\gamma-1} X \left(\frac{p_d}{p_a \gamma^\gamma X} \right)^{\frac{\gamma-1}{\gamma}} \\ &= T_a X^{1-\frac{\gamma-1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} = T_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}. \end{aligned} \quad (3)$$

Through D and a point E whose volume is greater than the original we have an isothermal

$$T_e = T_d \quad (4)$$

$$v_e = v_d \frac{p_d}{p_e} = v_a \left(\frac{p_a X}{p_d} \right)^{\frac{1}{\gamma}} \frac{p_d}{p_a}.$$

Hence

$$v_e = v_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \quad (5)$$

$$p_e = p_a. \quad (6)$$

Apply the perfect gas law to the points

$$\frac{p_a v_a}{T_a} = R$$

$$\frac{p_b v_b}{T_b} = \frac{p_a \tau_a \gamma^\gamma}{\gamma T_a \gamma^{\gamma-1}} = R$$

$$\frac{p_c v_c}{T_c} = \frac{p_a \gamma^\gamma X v_a}{T_a \gamma^{\gamma-1} X \gamma} = R$$

$$\frac{p_d v_d}{T_d} = \frac{p_d v_a p_a^{\frac{1}{\gamma}} X^{\frac{1}{\gamma}}}{T_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} p_a^{\frac{1}{\gamma}}} = R$$

$$\frac{p_e p_e}{T_e} = \frac{p_a v_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}}{T_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{1}{\gamma}}} = R.$$

During the isothermal compression heat must be abstracted, the amount can best be calculated by $O\varphi$ coördinates. Call this amount m , then,

$$m = T_d(\varphi_d - \varphi_e)$$

$$= T_e(\varphi_d - \varphi_e).$$

But

$$\varphi_d - \varphi_e = (\varphi_c - \varphi_b) - (\varphi_e - \varphi_a)$$

and

$$\varphi_e - \varphi_b = C_v \log_e \frac{T_c}{T_b} = C_v \log_e X,$$

$$\varphi_e - \varphi_a = C_p \log_e \frac{T_e}{T_a}$$

$$= C_p \log_e \left[X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right].$$

Besides this amount m we must abstract a quantity $C_p(T_e - T_a)$ isopiesticly, whence

$$\begin{aligned} H_2 &= C_p(T_e - T_a) + T_e \left\{ C_v \log_e X + C_p \log_e \left[X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\} \\ &= C_p T_a \left[X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right] + T_a X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left\{ C_p \log_e X \right. \\ &\quad \left. + C_p \log_e \left[X^{\frac{1}{\gamma}} \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}. \end{aligned} \quad (7)$$

$$W = H_1 - H_2, \quad (8)$$

$$E = 1 - \frac{H_1}{H_2}. \quad (9)$$

The volume range is

$$\begin{aligned} R_v &= v_d - v_b = v_a X^{\frac{1}{\gamma}} \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - \frac{v'_a}{\gamma} \\ &= v_a \left[X^{\frac{1}{\gamma}} \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right], \end{aligned} \quad (10)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = \frac{JW}{v_a \left[X^{\frac{1}{\gamma}} \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right]}, \quad (11)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{C_v \log_e X} \right), \quad (12)$$

$$R_p = p_c - p_d = p_a(\gamma)^{\gamma} X - p_d \quad (13)$$

$$\text{M.E.V.} = J \frac{W}{p_a(\gamma)^{\gamma} X - p_d}, \quad (14)$$

$$R_T = T_a((\gamma)^{\gamma-1} X - 1) \text{ as for II.} \quad (15)$$

CYCLE II. C.

Let Fig. 15 be the P.V. and Fig. 16 the $\theta\phi$ diagrams of the cycle.

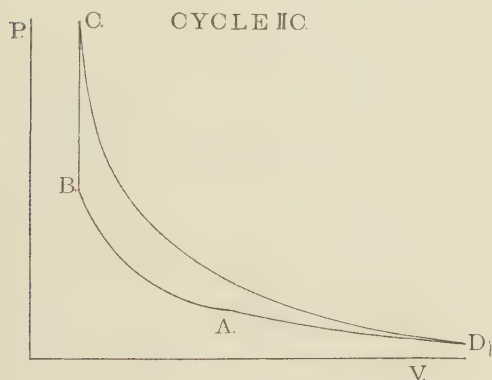


FIG. 15.

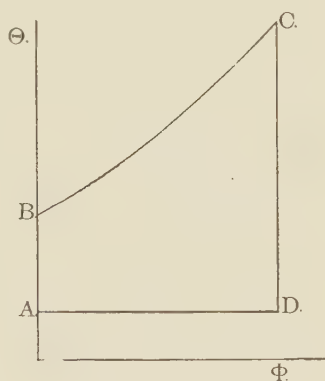


FIG. 16.

All values for the compression and heat addition found in Cycle II. may here be assumed. The point D lies at the intersection of two curves, one an adiabatic through C , the other an isothermal through A and the relations can be written. From the adiabatic relation

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}.$$

From the isothermal relation

$$v_d = v_a \frac{p_a}{p_d}.$$

Equating we get

$$v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = v_a \frac{p_a}{p_d}$$

$$\begin{aligned} \frac{v_a}{\gamma} \left(\frac{p_a \gamma X}{p_d} \right)^\gamma &= v_a \frac{p_a}{p_d} \\ \therefore p_d &= \frac{p_a}{X^{\frac{1}{\gamma-1}}} = p_a X^{\frac{1}{1-\gamma}}. \end{aligned} \quad (1)$$

This is the pressure at which the intersection will take place.

By substitution we get

$$v_d = v_a X^{\frac{1}{\gamma-1}} \quad (2)$$

$$T_d = T_a. \quad (3)$$

Applying the perfect gas law to D

$$\frac{p_d v_d}{T_d} = \frac{p_a v_a (X)^{\frac{1}{\gamma-1}}}{X^{\frac{1}{\gamma-1}} T_a} = \frac{p_a v_a}{T_a} = R.$$

All the heat is abstracted at constant temperature during the compression D to A . The entropy range is evidently the same as for heat addition and this is

$$R\phi = C_v \log_e X \quad (4)$$

whence

$$H_2 = T_a(\phi_d - \phi_a) = T_a C_v \log_e X \quad (5)$$

work

$$W = H_1 - H_2 = H_1 - T_a C_v \log_e X \quad (6)$$

$$E = 1 - \frac{H_2}{H_1} = 1 - \frac{T_a C_v \log_e X}{H_1} \quad (7)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R\phi} \right) = \frac{1}{2} \left(\frac{H_1 + T_a C_v \log_e X}{C_v \log_e X} \right)$$

whence

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1}{C_v \log_e X} - T_a \right) \quad (8)$$

$$R_v = v_d - v_b = v_a \left[X^{\frac{1}{\gamma-1}} - \frac{1}{\gamma} \right] \quad (9)$$

$$\text{M.E.P.} = \frac{JW}{R_v} = J \frac{H_1 - T_a C_v \log_e X}{v_a \left[X^{\frac{1}{\gamma-1}} - \frac{1}{\gamma} \right]} \quad (10)$$

$$R_p = p_c - p_d = p_a [\gamma^\gamma X - X^{\frac{1}{1-\gamma}}] \quad (11)$$

$$\text{M.E.V.} = \frac{JW}{R_p} = J \frac{H_1 - T_a C_v \log_v X}{p_a [\gamma^\gamma X - X^{\frac{1}{1-\gamma}}]} \quad (12)$$

$$R_T = T_a (\gamma^{\gamma-1} X - 1) \quad \text{as before} \quad (13)$$

Tabulate.

CYCLE II. C.

Symbol.	Formula as first derived.	Formula reduced.
p_b	$p_a \left(\frac{v'_a}{v'_b} \right)^\gamma$	$p_a \gamma^\gamma$
v_b	$\frac{v'_a}{\gamma}$	$\frac{v'_a}{\gamma}$
T_b	$T_a \left(\frac{v'_a}{v'_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
p_c	$p_b \frac{T_c}{T_b}$	$p_a \gamma^\gamma X$
v_c	v_b	$\frac{v'_a}{\gamma}$
T_c	$T_b \left(1 + \frac{H_1}{C_v T_b} \right)$	$T_a \gamma^{\gamma-1} X$
p_d	$p_a X^{1-\gamma}$	$p_a X^{1-\gamma}$
v_d	$v_a X^{\frac{1}{\gamma-1}}$	$v_a X^{\frac{1}{\gamma-1}}$
T_d	T_a	T_a
H_2	$T_a (\varphi_d - \varphi_a)$	$T_a C_v \log_e X$
W	$H_1 - H_2$	$H_1 - T_a C_v \log_e X$
E	$1 - \frac{H_2}{H_1}$	$\frac{1 - T_a C_v \log_e X}{H_1}$
R_ϕ	$C_v \log_e \frac{T_c}{T_b}$	$C_v \log_e X$
M.E.T.....	$\frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right)$	$\frac{1}{2} \left(\frac{H_1}{C_v \log_e X} - T_a \right)$

$$R_v \dots \dots \dots v_d - v_b \dots \dots \dots v_b \left(X^{\frac{1}{\gamma-1}} - \frac{1}{\gamma} \right)$$

$$\text{M.E.P.} \dots \dots \dots J \frac{W}{R_v} \dots \dots \dots J \left\{ \frac{H_1 - T_a C_v \log_e X}{v_a \left(X^{\frac{1}{\gamma-1}} - \frac{1}{\gamma} \right)} \right\}$$

$$R_p \dots \dots \dots p_c - p_d \dots \dots \dots p_a (\gamma^\gamma X - X^{\frac{1}{1-\gamma}})$$

$$\text{M.E.V.} \dots \dots \dots J \frac{W}{R_p} \dots \dots \dots J \left\{ \frac{H_1 - T_a C_v \log_e X}{p_a (\gamma^\gamma X - X^{\frac{1}{1-\gamma}})} \right\}$$

$$R_T \dots \dots \dots T_c - T_a \dots \dots \dots T_a (\gamma^{\gamma-1} X - 1)$$

CYCLE III.

Let Fig. 17 be its P.V. and Fig. 18 its $\theta\phi$ diagram.

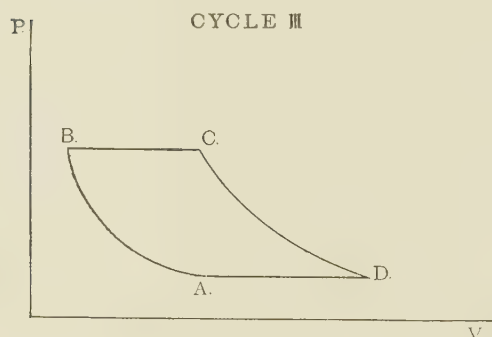


FIG. 17.

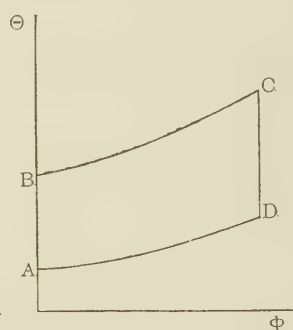


FIG. 18.

The compression results of Cycle II. may be assumed, hence

$$v_b = \frac{v_a}{\gamma}, \quad (1)$$

$$p_b = p_a \gamma^\gamma, \quad (2)$$

$$T_b = T_a \gamma^{\gamma-1}. \quad (3)$$

Heat is added isopiesticly, hence calling C_p the specific heat at constant pressure we have

$$T_c = T_b + \frac{H_1}{C_p} = T_b \left(1 + \frac{H_1}{C_p T_b} \right).$$

Write

$$1 + \frac{H_1}{\bar{C}_p T_b} = Y.$$

$$\therefore T_c = T_b Y = T_a \gamma^{\gamma-1} Y, \quad (4)$$

$$p_c = p_b = p_a \gamma^\gamma, \quad (5)$$

$$v_c = v_b \frac{T_c}{T_b} = v_b Y = \frac{v_a}{\gamma} Y. \quad (6)$$

Adiabatic expansion gives for final pressure of one atmosphere

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = \frac{v_a}{\gamma} Y \left(\frac{p_a \gamma^\gamma}{p_d} \right)^{\frac{1}{\gamma}} = v_a Y = \gamma v_c, \quad (7)$$

$$p_d = p_a \quad (8)$$

$$T_d = T_c \left(\frac{v_c}{v_d} \right)^{\gamma-1} = T_a \gamma^{\gamma-1} Y \left(\frac{v_a Y}{\gamma v_a} \right)^{\gamma-1}$$

$$\therefore T_d = T_a Y = \gamma^{\gamma-1} T_c. \quad (9)$$

Apply the perfect gas law

$$\frac{p_a v_a}{T_a} = R$$

$$\frac{p_b v_b}{T_b} = R \text{ as in II.}$$

$$\frac{p_c v_c}{T_c} = \frac{p_a \gamma^\gamma v_a Y}{T_a \gamma^{\gamma-1} Y \gamma} = R$$

$$\frac{p_d v_d}{T_d} = \frac{p_a v_a Y}{T_a Y} = R$$

Hence the formulæ are verified.

Heat is abstracted isopiastically

$$\begin{aligned} \therefore H_2 &= C_p (T_d - T_a) \\ &= C_p T_a (Y - 1) \end{aligned} \quad (10)$$

$$W = H_1 - H_2 = H_1 - C_p T_a (Y - 1) \quad (11)$$

$$E = \frac{W}{H_1} = 1 - \frac{H_2}{H_1} = 1 - \frac{C_p T_a (Y - 1)}{H_1} \quad (12)$$

Volume range is

$$R_v = v_d - v_b = v_a \left(Y - \frac{1}{\gamma} \right) \quad (13)$$

Whence for mean effective pressure

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{H_1 - C_p T_a (Y - 1)}{v_a \left(Y - \frac{1}{\gamma} \right)} \quad (14)$$

$$R\phi = \phi_c - \phi_b = C_p \log_e \frac{T_c}{T_b} = C_p \log_e Y. \quad (15)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R\phi} \right) = \frac{1}{2} \left(\frac{H_1 + C_p T_a (Y - 1)}{C_p \log_e Y} \right) \quad (16)$$

$$R_p = p_b - p_a = p_a (\gamma^\gamma - 1) \quad (17)$$

$$\text{M.E.V.} = J \frac{W}{R_p} = J \frac{H_1 - C_p T_a (Y - 1)}{p_a (\gamma^\gamma - 1)} \quad (18)$$

$$R_T = T_c - T_a = T_a (\gamma^{\gamma-1} Y - 1). \quad (19)$$

Tabulate.

CYCLE III.

Symbol.	Formula as derived.	Formula reduced.
p_b	$p_a \left(\frac{v_a}{v_b} \right)^\gamma$	$p_a \gamma^\gamma$
v_b	$\frac{v_a}{\gamma}$	$\frac{v_a}{\gamma}$
T_b	$T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
p_c	p_b	$p_a \gamma^\gamma$
v_c	$v_b \frac{T_c}{T_b}$	$\frac{v_a}{\gamma} Y$
T_c	$T_b \left(1 + \frac{H_1}{C_p T_b} \right)$	$T_a \gamma^{\gamma-1} Y$
p_d	p_a	p_a
v_d	$v_c \left(\frac{p_c}{p_a} \right)^\frac{1}{\gamma}$	$v_a Y$
T_d	$T_c \left(\frac{v_c}{v_d} \right)^{\gamma-1}$	$T_a Y$

$$\begin{aligned}
H_2 &\dots\dots\dots C_p(T_d - T_a) \dots\dots\dots C_p T_a(Y - 1) \\
W &\dots\dots\dots H_1 - H_2 \dots\dots\dots H_1 - C_p T_a(Y - 1) \\
E &\dots\dots\dots 1 - \frac{H_2}{H_1} \dots\dots\dots 1 - \frac{C_p T_a(Y - 1)}{H_1} \\
R\phi &\dots\dots\dots C_p \log_e \frac{T_c}{T_b} \dots\dots\dots C_p \log_e Y \\
\text{M.E.T.} &\dots\dots\dots \frac{1}{2} \left(\frac{H_1 + H_2}{R\phi} \right) \dots\dots\dots \frac{1}{2} \left\{ \frac{H_1 + C_p T_a(Y - 1)}{C_p \log_e Y} \right\} \\
R_v &\dots\dots\dots v_d - v_b \dots\dots\dots v_a \left(Y - \frac{1}{\gamma} \right) \\
\text{M.E.P.} &\dots\dots\dots \int \frac{W}{R_v} \dots\dots\dots \int \left\{ \frac{H_1 - C_p T_a(Y - 1)}{v_a \left(Y - \frac{1}{\gamma} \right)} \right\} \\
R_p &\dots\dots\dots p_b - p_a \dots\dots\dots p_b(\gamma^\gamma - 1) \\
\text{M.E.V.} &\dots\dots\dots \int \frac{W}{R_p} \dots\dots\dots \int \left\{ \frac{H_1 - C_p T_a(Y - 1)}{p_a(\gamma^\gamma - 1)} \right\} \\
R_T &
\end{aligned}$$

CYCLE III. A.

Fig. 19 is its P.V. and Fig. 20 its $\theta\phi$ diagram.

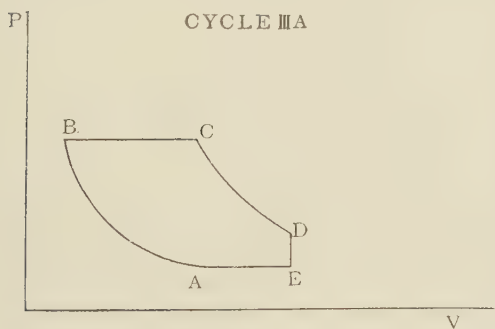


FIG. 19.

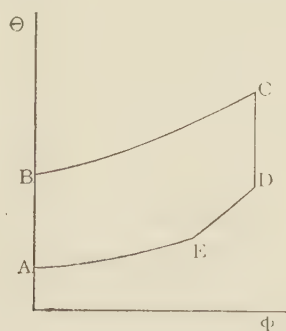


FIG. 20.

Assume the results of III. up to point C. The point D is situated anywhere on the adiabatic through C between C and atmosphere.

Write

$$p_c > p_d > p_a \quad (1)$$

and

$$v_d > v_a. \quad (2)$$

This latter (2) will not necessarily follow from (1) but where it does not hold the cycle is decidedly imperfect and this case is here neglected, *i. e.*, the case where the isometric DE cuts the adiabatic AB .

We have then

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = \frac{v_a}{\gamma} Y \left(\frac{p_a \gamma^{\gamma}}{p_d} \right)^{\frac{1}{\gamma}} = v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \quad (3)$$

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} = T_a \gamma^{\gamma-1} Y \left(\frac{p_d}{p_a \gamma^{\gamma}} \right)^{\frac{\gamma-1}{\gamma}} = T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \quad (4)$$

$$p_e = p_a \quad (5)$$

$$v_e = v_d = v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \quad (6)$$

$$T_e = T_a \frac{v_e}{v_a} = T_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}. \quad (7)$$

Apply the perfect gas law to D and E .

$$\frac{p_d v_d}{T_d} = \frac{p_d v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}}{T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}} = R$$

$$\frac{p_e v_e}{T_e} = \frac{p_a v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}}{T_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}} = R.$$

This verifies the formulæ.

Heat is abstracted in two parts and the amount is

$$H_2 = C_v(T_d - T_e) + C_p(T_e - T_a) \quad (8)$$

$$\begin{aligned} &= C_v \left[T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - T_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \right] \\ &+ C_p \left[T_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - T_a \right] \end{aligned} \quad (9)$$

$$= C_v T_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \left[\frac{p_d}{p_a} - 1 \right] + C_p T_a \left[Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - 1 \right].$$

The work done is

$$W = H_1 - H_2 \quad (10)$$

and efficiency

$$E = 1 - \frac{H_2}{H_1} \quad (11)$$

$$R_v = v_d - v_b = v_a \left[Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - 1 \right] \quad (12)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{W}{v_a \left[Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - 1 \right]} \quad (13)$$

$$R\phi = C_p \log_e Y \text{ as before for III.} \quad (14)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_v} \right) = \frac{1}{2} \left(\frac{H_1 + H_2}{C_p \log_e Y} \right) \quad (15)$$

$$R_p = p_c - p_a = p_a (\gamma^\gamma - 1) \text{ as in III.} \quad (16)$$

$$\therefore \text{M.E.V.} = J \frac{W}{R_p} = J \frac{W}{p_a (\gamma^\gamma - 1)}. \quad (17)$$

As before III. the temperature range

$$R_T = T_e - T_a = T_a (\gamma^{\gamma-1} Y - 1) \quad (18)$$

CYCLE III. B.

Figs. 21 and 22 are its diagrams

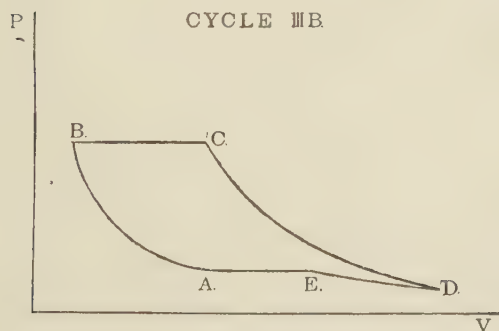


FIG. 21

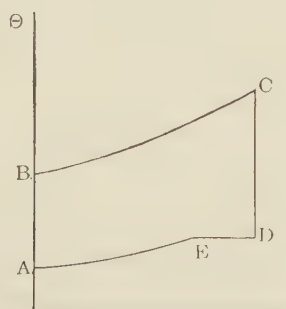


FIG. 22.

All results of III. A up to period D may be assumed except that p_d which was there arbitrary was assumed greater than p_a is here less than p_a , i. e.,

$$p_e > p_d > 0. \quad (1)$$

We had

$$v_d = v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}, \quad (2)$$

and

$$T_d = T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}. \quad (3)$$

Through E and D there must pass an isothermal and

$$v_e > v_a \quad (4)$$

$$p_e = p_a \quad (5)$$

$$v_e = v_d \frac{p_d}{p_e} = v_d \frac{p_d}{p_a} = v_a Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \frac{p_d}{p_a}$$

$$\therefore v_e = v_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \quad (6)$$

$$T_e = T_d = T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}. \quad (7)$$

Applying the perfect gas law to E

$$\frac{p_e v_e}{T_e} = \frac{p_a v_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}}{T_a Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}}}. \quad (8)$$

Heat abstracted 1° isothermally a quantity m .

2° isopiesticly “ “ n .

$$n = C_p (T_e - T_a)$$

$$= C_p T_a \left[Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right],$$

$$m = T_e (\varphi_d - \varphi_e).$$

But

$$\varphi_d - \varphi_e = (\varphi_b - \varphi_c) - (\varphi_e - \varphi_a),$$

$$= C_p \log_e \frac{T_c}{T_b} = C_p \log_e \frac{T_e}{T_a}.$$

$$= C_p \log_e Y - C_p \log_e Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}},$$

$$= C_p \log_e Y \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right].$$

$$\therefore H_2 = C_p T_a \left\{ Y \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 + Y \log_e Y \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}, \quad (9)$$

$$W = H_1 - H_2, \quad (10)$$

$$E = 1 - \frac{H_2}{H_1},$$

$$R_v = v_d - v_b = v_a \left[Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right], \quad (13)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{W}{v_d \left[Y \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} - \frac{1}{\gamma} \right]}, \quad (14)$$

$$R_p = p_c - p_d = p_a \gamma^{\gamma} - p_d, \quad (15)$$

$$\text{M.E.V.} = J \frac{W}{p_a \gamma^{\gamma} - p_d}, \quad (16)$$

$$R_T = T_c - T_a = T_a (\gamma^{\gamma-1} Y - 1) \quad \text{as before III.} \quad (17)$$

CYCLE III. C.

Let Figs. 23 and 24 be its diagrams.

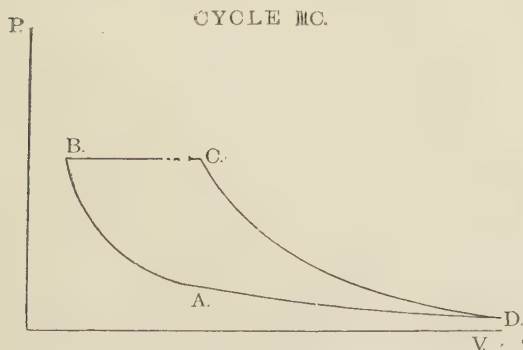


FIG. 23.

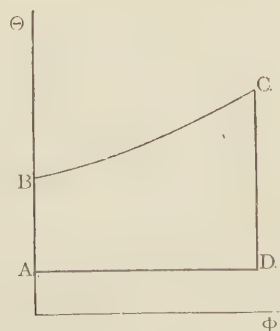


FIG. 24.

We will assume all results to C already derived. The point D is determined by the intersection of the adiabatic through C with the isothermal through A . From the adiabatic relation

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}}.$$

From the isothermal relation

$$v_d = v_a \frac{p_a}{p_d}$$

$$\therefore v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = v_a \frac{p_a}{p_d} = \frac{v_a Y}{\gamma} \left(\frac{p_a \gamma}{p_d} \right)^{\frac{1}{\gamma}}$$

$$\therefore p_d = \frac{p_a}{Y^{\frac{\gamma}{\gamma-1}}}. \quad (1)$$

By substitution

$$v_d = v_a Y^{\frac{\gamma}{\gamma-1}} \quad (2)$$

$$T_d = T_a \quad (3)$$

$$H_2 = T_d(\varphi_c - \varphi_b) = T_a C_p \log_e Y \quad (4)$$

$$W = H_1 - H_2 = H_1 - T_a C_p \log_e Y \quad (5)$$

$$E = 1 - \frac{T_a C_p \log_e Y}{H_1}$$

$$R_v = v_d - v_b = v_a \left[Y^{\frac{\gamma}{\gamma-1}} - \frac{1}{\gamma} \right] \quad (6)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{H_1 - T_a C_p \log_e Y}{v_a \left(Y^{\frac{\gamma}{\gamma-1}} - \frac{1}{\gamma} \right)} \quad (7)$$

$$R_p = p_b - p_d = p_a \left(\gamma^{\gamma} - \frac{1}{Y^{\frac{\gamma}{\gamma-1}}} \right) \quad (8)$$

$$\text{M.E.V.} = J \frac{W}{R_p} = J \frac{H_1 - T_a C_p \log_e Y}{p_a \left(\gamma^{\gamma} - \frac{1}{Y^{\frac{\gamma}{\gamma-1}}} \right)} \quad (9)$$

$$\begin{aligned} \text{M.E.T.} &= \frac{1}{2} \left(\frac{H_1 + H_2}{R\varphi} \right) = \frac{1}{2} \left(\frac{H_1 - T_a C_p \log_e Y}{C_p \log_e Y} \right) \\ &= \frac{1}{2} \left(\frac{H_1}{C_p \log_e Y} + T_a \right) \end{aligned} \quad (10)$$

$$R_T = T_c - T_a = T_a [\gamma^{\gamma-1} Y - 1] \text{ as in III.} \quad (11)$$

CYCLE III. C.

Symbol.	Formula as First Derived.	Formula Reduced.
p_b	$p_a \left(\frac{v_a}{v_b} \right)^\gamma$	$p_a \gamma^\gamma$
v_b	$\frac{v_a}{\gamma}$	$\frac{v_a}{\gamma}$
T_b	$T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
p_c	p_b	$p_a \gamma^\gamma$
v_c	$v_b \frac{T_c}{T_b}$	$\frac{v_a}{\gamma} Y$
T_c	$T_b \left(1 + \frac{H_1}{C_p T_b} \right)$	$T_a \gamma^{\gamma-1} Y$
p_d		$p_a Y^{\frac{\gamma}{\gamma-1}}$
v_d		$\frac{v_a}{\gamma} Y^{\frac{\gamma}{\gamma-1}}$
T_d		T_a
H_2	$T_d(\varphi_c - \varphi_b)$	$T_a C_p \log_e Y$
W	$H_1 - H_2$	$H_1 - T_a C_p \log_e Y$
E	$1 - \frac{H_2}{H_1}$	$1 - \frac{T_a C_p \log_e Y}{H_1}$
$R\varphi$	$C_p \log_e \frac{T_c}{T_b}$	$C_p \log_e Y$
M.E.T.....	$\frac{1}{2} \left(\frac{H_1 + H_2}{R\varphi} \right)$	$\frac{1}{2} \left(\frac{H_1}{C_p \log_e Y} + T_a \right)$

$$R_v \dots \dots \dots v_d - v_b \dots \dots \dots v_a \left(Y^{\frac{\gamma}{\gamma-1}} - \frac{1}{\gamma} \right)$$

$$\text{M.E.P.} \dots \dots \dots J \frac{W}{R_v} \dots \dots \dots J \left\{ \frac{H_1 - T_a C_p \log_e Y}{v_a \left(Y^{\frac{\gamma}{\gamma-1}} - \frac{1}{\gamma} \right)} \right\}$$

$$R_p \dots \dots \dots p_b - p_a \dots \dots \dots p_a \left(\gamma^\gamma - \frac{1}{Y^{\frac{\gamma}{\gamma-1}}} \right)$$

$$\text{M.E.V.} \dots \dots \dots J \frac{W}{R_p} \dots \dots \dots J \left\{ \frac{H_1 - T_a C_p \log_e Y}{p_a (\gamma^\gamma - Y^{1-\gamma})} \right\}$$

$$R_T \dots \dots \dots T_c - T_a \dots \dots \dots T_a (\gamma^{\gamma-1} Y - 1)$$

CYCLE IV.

Figs. 25 and 26 are its diagrams.

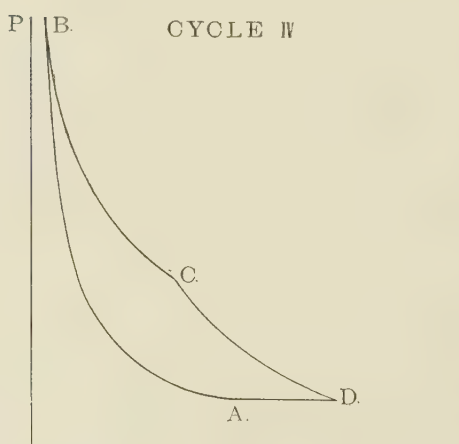


FIG. 25.

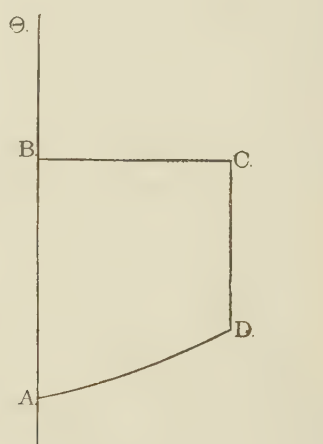


FIG. 26.

We may assume the results already obtained for the compression but beyond that new conditions arise. By isothermal heating the curve approaches the atmospheric line and there will be a certain quantity of heat that will bring the isothermal down to the atmospheric line leaving a subsequent adiabatic expansion an impossibility. This quantity of course depends on the location of B, *i. e.*, the amount of previous compression. The higher the pre-

vious compression the more heat may we add isothermally before reaching atmospheric pressure.

The quantity of heat which will make adiabatic expansion impossible and stop the isothermal on the atmospheric line can best be determined from $\theta\phi$ relations. Denote this quantity by Q .

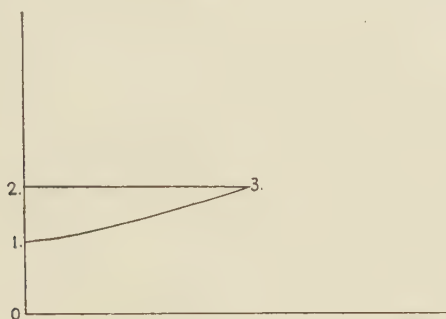


FIG. 45.

On the $\theta\phi$ diagram Fig. 45 the point 3 lies at the intersection of the isothermal 23 drawn at temperature $O2$ the compression temperature and the isopiestic 13 drawn from atmospheric temperature $O1$ to the intersection 3. In each case the entropy range is

$$\varphi_3 - \varphi_1 = C_p \log_e \frac{T_3}{T_1} = \varphi_3 - \varphi_2$$

$$\therefore Q = T_2 C_p \log_e \frac{T_2}{T_1}$$

Apply now to the Cycle IV.

$$\begin{aligned} \therefore Q &= T_b C_p \log_e \frac{T_b}{T_a} \\ &= T_a \gamma^{\gamma-1} \log_e \gamma^{\gamma-1}. \end{aligned} \quad (1)$$

This is the amount of heat that will bring C down to atmosphere with no adiabatic expansion. In order that the cycle may exist according to the hypothetical definition we must add less heat than this quantity Q . Hence we have the equation of condition for the existence of the cycle

$$H_1 < T_a \gamma^{\gamma-1} \log_e \gamma^{\gamma-1} \quad (2)$$

or

$$H_1 < T_b \log_e \frac{T_b}{T_a}$$

A similar method can be used to find the amount of expansion or resulting pressure and volume after addition of H_1 , BTU of heat.

Draw on both diagrams the isopiestic through the termination C of the isothermal and cutting the adiabat AB at point c' .

Then

$$\begin{aligned} T_{c'} &= T_a \left(\frac{p_{c'}}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \\ &= T_a \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}}, \\ \varphi_c - \varphi_{c'} &= C_p \log_e \frac{T_c}{T_{c'}} = C_p \log_e \frac{T_b}{T_{c'}} \\ &= C_p \log_e \frac{T_a \gamma^{\gamma-1}}{T_a \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}}}. \end{aligned}$$

But

$$\begin{aligned} \gamma^{\gamma-1} &= \left(\frac{v_a}{v_b} \right)^{\gamma-1} = \left(\frac{p_b}{p_a} \right)^{\frac{\gamma-1}{\gamma}}. \\ \therefore \varphi_c - \varphi_{c'} &= C_p \log_e \left(\frac{p_b p_a}{p_a p_c} \right)^{\frac{\gamma-1}{\gamma}} \\ &= C_p \log_e \left(\frac{p_b}{p_c} \right)^{\frac{\gamma-1}{\gamma}}. \end{aligned}$$

And the amount of heat necessary for this isothermal expansion from B to C

$$H_1 = T_b C_p \log_e \left(\frac{p_b}{p_c} \right)^{\frac{\gamma-1}{\gamma}}.$$

But

$$\frac{\gamma-1}{\gamma} = \frac{\frac{C_p}{C_v} - 1}{\frac{C_p}{C_v}} = \frac{C_p - C_v}{C_p}.$$

$$\therefore H_1 = T_b (C_p - C_v) \log_e \frac{p_b}{p_c}$$

and

$$\log_e \frac{p_b}{p_c} = \frac{H_1}{T_b (C_p - C_v)}.$$

Put

$$T_b(C_p - C_v) = Z.$$

Then will

$$\frac{p_b}{p_c} = e^Z,$$

$$p_c = \frac{p_b}{e^Z}.$$

That is to say if we start at state B and add a quantity of heat H_1 , isothermally the resulting pressure is

$$p_c = \frac{p_b}{e^Z} = \frac{p_a \gamma^\gamma}{e^Z}. \quad (3)$$

Since

$$\frac{p_b}{p_c} = \frac{v_c}{v_b},$$

$$\frac{v_c}{v_b} = e^Z,$$

$$v_c = v_b e^Z = \frac{v_a}{\gamma} e^Z, \quad (4)$$

$$T_c = T_b = T_a \gamma^{\gamma-1}, \quad (5)$$

$$p_d = p_a, \quad (6)$$

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = \frac{v_a e^Z}{\gamma} \left(\frac{p_a \gamma^\gamma}{e^Z p_a} \right)^{\frac{1}{\gamma}} = v_a e^{Z \left(\frac{\gamma-1}{\gamma} \right)}.$$

Now

$$Z = T_b(C_p - C_v) \frac{H_1}{T_b(C_p - C_v)}$$

$$\therefore Z \left(\frac{\gamma-1}{\gamma} \right) = \frac{H_1(C_p - C_v)}{T_b C_p (C_p - C_v)} = \frac{H_1}{T_b C_p}.$$

We had III.,

$$Y = 1 + \frac{H_1}{T_b C_p}.$$

Hence

$$Z \left(\frac{\gamma-1}{\gamma} \right) = Y - 1.$$

Whence

$$v_d = v_a e^{Y-1}. \quad (7)$$

Similarly

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} = T_b \left(\frac{p_a e^Z}{p_a \gamma^\gamma} \right)^{\frac{\gamma-1}{\gamma}} = T_a e^{Z \left(\frac{\gamma-1}{\gamma} \right)} = T_a e^{Y-1}. \quad (8)$$

Apply the perfect gas law

$$\frac{p_a v_a}{T_a} = R,$$

$$\frac{p_b v_b}{T_b} = \frac{v_a p_a \gamma^\gamma}{\gamma T_a \gamma^{\gamma-1}} = R.$$

$$\frac{p_c v_c}{T_c} = \frac{p_a \gamma^\gamma v_a e^Z}{e^Z \gamma T_a \gamma^{\gamma-1}} = R.$$

$$\frac{p_d v_d}{T_d} = \frac{p_a v_a e^{Y-1}}{T_a e^{Y-1}} = R.$$

Verifying the formulæ,

$$H_2 = C_p(T_d - T_a) = C_p T_a (e^{Y-1} - 1) \quad (9)$$

$$W = H_1 - H_2 = H_1 - C_p T_a (e^{Y-1} - 1) \quad (10)$$

$$E = \frac{W}{H_1} = 1 - \frac{C_p T_a (e^{Y-1} - 1)}{H_1} \quad (11)$$

$$R_\phi = \frac{H_1}{T_b} \quad (12)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) = \frac{1}{2} \left\{ H_1 + \frac{C_p T_a (e^{Y-1} - 1)}{\frac{H_1}{T_b}} \right\}.$$

$$\begin{aligned} \therefore \text{M.E.T.} &= \frac{1}{2} \left[T_b + \frac{C_p T_a T_b (e^{Y-1})}{H_1} \right] \\ &= \frac{1}{2} \left[T_a \gamma^{Y-1} + \frac{C_p (T_a)^2 \gamma^{Y-1} (e^{Y-1} - 1)}{H_1} \right], \end{aligned} \quad (13)$$

$$R_v = v_d - v_b = v_a \left[e^{Y-1} - \frac{1}{\gamma} \right], \quad (14)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{H_1 - C_p T_a (e^{Y-1} - 1)}{v_a \left(e^{Y-1} - \frac{1}{\gamma} \right)}, \quad (15)$$

$$R_p = p_b - p_a = p_a(\gamma^\gamma - 1), \quad (16)$$

$$\text{M.E.V.} = J \frac{W}{R_p} = J \frac{H_1 - C_p T_a (e^{\gamma-1} - 1)}{p_a (\gamma^\gamma - 1)}, \quad (17)$$

$$R_T = T_b - T_a = T_a(\gamma^{\gamma-1} - 1). \quad (18)$$

CYCLE IV.

Symbol.	Formula as Derived.	Formula Reduced.
p_b	$p_a \left(\frac{v_a}{v_b} \right)^\gamma$	$p_a \gamma^\gamma$
v_b	$\frac{v_a}{\gamma}$	$\frac{v_a}{\gamma}$
T_b	$T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
Equation of condition...	$H_1 < T_b \log_e \frac{T_b}{T_a} \dots H_1 < T_a \gamma^{\gamma-1} \log_e \gamma^{\gamma-1}$	
p_c	$\frac{p_b}{\frac{H_1}{e^{T_b(C_p - C_v)}}}$	$\frac{p_a \gamma^\gamma}{e^Z}$
v_c	$v_b \frac{p_b}{p_c}$	$\frac{v_a}{\gamma} e^Z$
T_c	T_b	$T_a \gamma^{\gamma-1}$
p_d	p_a	p_a
v_d	$v_c \left(\frac{p_c}{p_d} \right)^\gamma$	$v_a e^{\gamma-1}$
T_d	$T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}}$	$T_a e^{\gamma-1}$
H_2	$C_p(T_d - T_a)$	$C_p T_a (e^{\gamma-1} - 1)$
W	$H_1 - H_2$	$H_1 - C_p T_a (e^{\gamma-1} - 1)$
E	$1 - \frac{H_2}{H_1}$	$1 - C_p T_a (e^{\gamma-1} - 1)$
R_ϕ	$\frac{H_1}{T_b}$	$\frac{H_1}{\gamma^{\gamma-1} T_a}$

$$\begin{aligned}
 \text{M.E.T.} & \dots\dots\dots \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) \dots\dots \\
 & \qquad \qquad \qquad \frac{\gamma^{\gamma-1} T_a}{2} \left[\frac{H_1 + C_p T_a (e^{\gamma-1} - 1)}{H_1} \right] \\
 R_v & \dots\dots\dots v_d - v_b \dots\dots\dots v_a \left(e^{\gamma-1} - \frac{1}{\gamma} \right) \\
 \text{M.E.P.} & \dots\dots\dots J_{R_v}^W \dots\dots\dots J \left\{ \frac{H_1 - C_p T_a (e^{\gamma-1} - 1)}{v_a \left(e^{\gamma-1} - \frac{1}{\gamma} \right)} \right\} \\
 R_p & \dots\dots\dots p_b - p_a \dots\dots\dots p_a (\gamma^\gamma - 1) \\
 \text{M.E.V.} & \dots\dots\dots J_{R_p}^W \dots\dots\dots J \left[\frac{H_1 - C_p T_a (e^{\gamma-1} - 1)}{p_a (\gamma^{\gamma-1} - 1)} \right] \\
 R_T & \dots\dots\dots T_b - T_a \dots\dots\dots T_a (\gamma^{\gamma-1} - 1)
 \end{aligned}$$

CYCLE IV. A.

Figs. 27 and 28 are its diagrams.

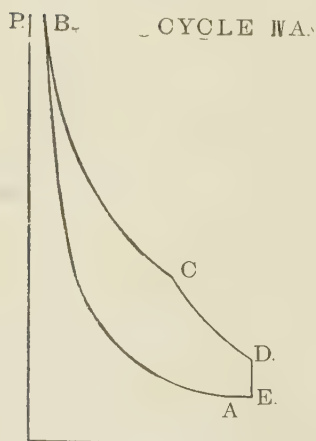


FIG. 27.

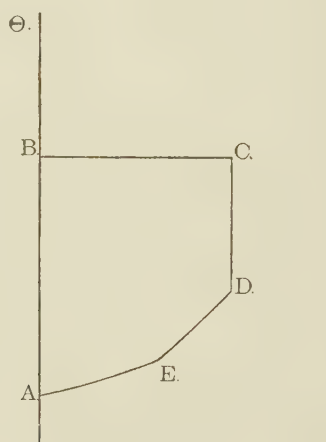


FIG. 28.

We may assume the results of IV. up to point C. The point D lies somewhere on the adiabat between C and atmosphere and is subject to the conditions

$$p_c > p_d > p_e \quad (1)$$

Then
$$v_d > v_a. \quad (2)$$

$$v_d = v_c \left(\frac{p_c}{p_d} \right)^{\frac{1}{\gamma}} = \frac{v_a}{\gamma} e^Z \left(\frac{p_a \gamma^\gamma}{p_a \gamma^Z} \right)^{\frac{1}{\gamma}} = v_a e^{Z(\frac{\gamma-1}{\gamma})} \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}}.$$

$$\therefore v_d = v_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1}. \quad (3)$$

Similarly

$$T_d = T_c \left(\frac{p_d}{p_c} \right)^{\frac{\gamma-1}{\gamma}} = T_a \gamma^{\gamma-1} \left(\frac{p_d e^Z}{p_a \gamma^\gamma} \right)^{\frac{1}{\gamma}}$$

$$\therefore T_d = T_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \quad (4)$$

$$v_e = v_d = v_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1} \quad (5)$$

$$p_e = p_a$$

$$T_e = T_d \frac{p_e}{p_d} = T_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \frac{p_a}{p_d} = T_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1}.$$

Apply the perfect gas law to *D* and *E*.

$$\frac{p_d v_d}{T_e} = \frac{p_d v_a p_a^{\frac{1}{\gamma}} e^{Y-1} p_a^{\frac{\gamma-1}{\gamma}}}{p_a^{\frac{1}{\gamma}} T_a p_d^{\frac{\gamma-1}{\gamma}} e^{Y-1}} = R$$

$$\frac{p_e v_e}{T_e} = \frac{p_a v_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1}}{T_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1}} = R.$$

The heat abstracted is

$$H_2 = C_v(T_d - T_e) + C_p(T_e - T_a)$$

$$= C_v \left[T_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} - T_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1} \right]$$

$$+ C_p \left[T_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1} - T_a \right]$$

$$\therefore H_2 = C_v T_a e^{Y-1} \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} \left[\frac{p_d}{p_a} - 1 \right] - C_p T_a \left[\left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1} - 1 \right] \quad (6)$$

$$W = H_1 - H_2 \quad (7)$$

$$E = 1 - \frac{H_2}{H_1} \quad (8)$$

$$R_v = v_d - v_a = v_a \left[\left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{\gamma-1} - \frac{1}{\gamma} \right] \quad (9)$$

$$\text{M.E.P.} = J \frac{W}{v_a \left[\left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{\gamma-1} - \frac{1}{\gamma} \right]} \quad (10)$$

$$R_p = p_b - p_a = p_a(\gamma^\gamma - 1)$$

$$\text{M.E.V.} = J \frac{W}{p_a(\gamma^\gamma - 1)} \quad (11)$$

$$R_\phi = \frac{H_1}{T_b} \quad (12)$$

$$\text{M.E.T.} = \frac{1}{2} \left\{ \frac{H_1 + H_2}{\frac{H_1}{T_b}} \right\} = \frac{T_b}{2} \left(1 - \frac{H_2}{H_1} \right) \quad (13)$$

$$R_T = T_b - T_a = T_a(\gamma^{\gamma-1} - 1). \quad (14)$$

CYCLE IV. B.

Let Figs. 29 and 30 be its diagrams.

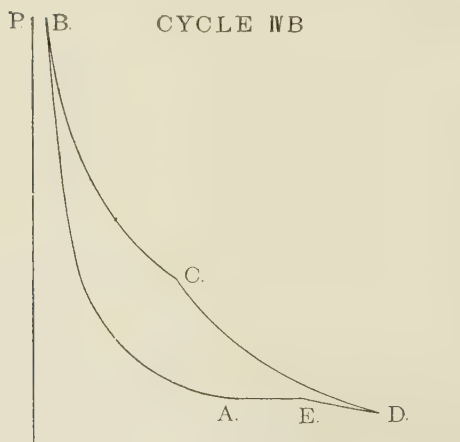


FIG. 29.

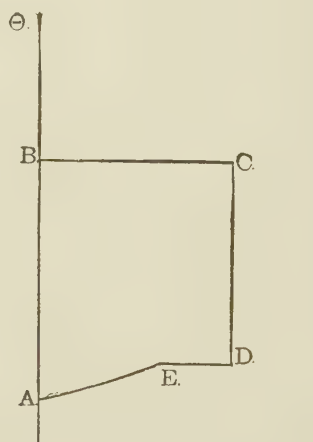


FIG. 30.

FIG. 29.

FIG. 30.

The operations up to C are as in IV. and we may assume those results.

The point D is subject to the condition

$$p_d < p_a \quad (1)$$

and the point E to the condition

$$v_e > v_a. \quad (2)$$

Then

$$v_d = v_a \left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma}} e^{Y-1} \quad (3)$$

and

$$T_d = T_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \quad (4)$$

$$T_e = T_d = T_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \quad (5)$$

$$p_e = p_a$$

$$v_e = v_d \frac{p_d}{p_e} = v_a \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1}. \quad (6)$$

Following the methods already adopted we can write

$$H_2 = T_d(\varphi_d - \varphi_e) + C_p(T_e - T_a).$$

But

$$\varphi_d - \varphi_e = (\varphi_c - \varphi_b) - (\varphi_e - \varphi_a)$$

$$\varphi_c - \varphi_b = \frac{H_1}{T_b} = \frac{H_1}{T_a e^{Y-1}}$$

$$\varphi_e - \varphi_a = C_p \log_e \frac{T_e}{T_a}$$

$$= C_p \log_e \left[\left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \right],$$

$$\therefore H_2 = T_e \left[\frac{H_1}{T_a e^{Y-1}} - C_p \log_e \left\{ \left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} \right\} \right] \quad (7)$$

$$+ C_p T_a \left[\left(\frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} e^{Y-1} - 1 \right]$$

$$W = H_1 - H_2 \quad (8)$$

$$E = 1 - \frac{H_2}{H_1} \quad (9)$$

$$R_r = v_d - v_b = v_a \left[\left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma} c^{r-1}} - \frac{1}{\gamma} \right]. \quad (10)$$

$$\therefore \text{M.E.P.} = J \frac{W}{v_a \left[\left(\frac{p_a}{p_d} \right)^{\frac{1}{\gamma} c^{r-1}} - \frac{1}{\gamma} \right]} \quad (11)$$

$$R_p = p_b - p_d = p_a \gamma^r - p_d \quad (12)$$

$$\therefore \text{M.E.V.} = J \frac{W}{p_a \gamma^r - p_d} \quad (13)$$

$$R_\phi = \frac{H_1}{T_b} = \frac{H_1}{T_a \gamma^r} \quad (14)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) = \frac{1}{2} \left\{ \frac{H_1 + H_2}{\frac{H_1}{T_b}} \right\} \quad (15)$$

$$= \frac{1}{2} \left(T_b + T_b \frac{H_2}{H_1} \right) = \frac{T_b}{2} \left(1 + \frac{H_2}{H_1} \right)$$

$$R_T = T_b - T_a = T_a (\gamma^{r-1} - 1). \quad (16)$$

CYCLE IV. C.

Let Figs. 31 and 32 be its diagrams.

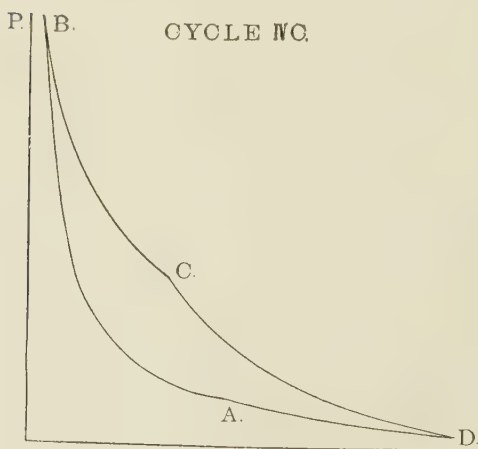


FIG. 31.

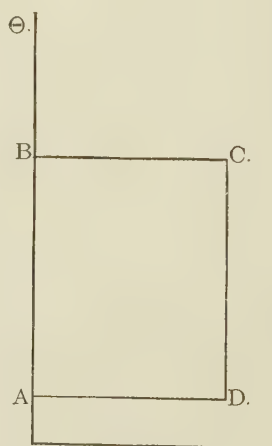


FIG. 32.

Assume results up to C as in IV.

The adiabatic through C must meet the isothermal through A to locate the point D .

From the adiabatic relations

$$p_d = p_c \left(\frac{v_c}{v_d} \right)^\gamma.$$

From the isothermal relation

$$p_d = p_a \frac{v_a}{v_d}.$$

$$\therefore p_c \left(\frac{v_c}{v_d} \right)^\gamma = p_a \frac{v_a}{v_d}$$

$$p_c v_c^\gamma = p_a v_a v_d^{\gamma-1}.$$

$$\therefore v_d = \left(\frac{p_c v_c^\gamma}{p_a v_a} \right)^{\frac{1}{\gamma-1}} = \left(\frac{p_a \gamma v_a^\gamma e^{\gamma Z}}{e^Z \gamma p_a v_a} \right)^{\frac{1}{\gamma-1}}.$$

$$\therefore v_d = v_a e^Z = \gamma v_c. \quad (1)$$

By substitution

$$p_d = \frac{p_a}{e^Z} = \frac{p_c}{\gamma^\gamma} \quad (2)$$

$$T_d = T_a. \quad (3)$$

By inspection it is easily seen the perfect gas law is satisfied.

$$H_2 = T_a(\varphi_d - \varphi_a) = T_a(\varphi_c - \varphi_b) = T_a \frac{H_1}{T_b} = \frac{T_a H_1}{T_a \gamma^{\gamma-1}}.$$

$$\therefore H_2 = \frac{H_1}{\gamma^{\gamma-1}} \quad (4)$$

$$W = H_1 - H_2 = H_1 \left(1 - \frac{1}{\gamma^{\gamma-1}} \right) \quad (5)$$

$$E = \frac{W}{H_1} = 1 - \frac{H_2}{H_1} = 1 - \frac{1}{\gamma^{\gamma-1}} \quad (6)$$

$$R_v = v_d - v_b = v_a \left(e^Z - \frac{1}{\gamma} \right) \quad (7)$$

$$\text{M.E.P.} = J \frac{H_1 \left(1 - \frac{1}{\gamma^{\gamma-1}} \right)}{v_a \left(e^Z - \frac{1}{\gamma} \right)} \quad (8)$$

$$R_p = p_b - p_d = p_a \left(\gamma^\gamma - \frac{1}{e^Z} \right) \quad (9)$$

$$\text{M.E.V.} = J \frac{H_1 \left(1 - \frac{1}{\gamma^{\gamma-1}} \right)}{p_a \left(\gamma^\gamma - \frac{1}{e^Z} \right)}$$

$$\text{M.E.T.} = \frac{1}{2}(T_a + T_b) = \frac{T_a}{2}(1 + \gamma^{\gamma-1}). \quad (10)$$

CYCLE IV.—C.

Symbol.	Formula as Derived.	Formula Reduced.
---------	---------------------	------------------

p_b	$p_a \left(\frac{v_a}{v_b} \right)^\gamma$	$p_a \gamma^\gamma$
-------	---	---------------------

v_b	$\frac{v_a}{\gamma}$	$\frac{v_a}{\gamma}$
-------	----------------------	----------------------

T_b	$T_a \left(\frac{v_a}{v_b} \right)^{\gamma-1}$	$T_a \gamma^{\gamma-1}$
-------	---	-------------------------

Equation of condition	$H_1 > 0$	$H_1 > 0$
-----------------------	-----------	-----------

p_c	$\frac{p_b}{\frac{H_1}{e^{T_b(C_p - C_v)}}}$	$\frac{p_a \gamma^\gamma}{e^Z}$
-------	--	---------------------------------

v_c	$v_b \frac{p_b}{p_c}$	$\frac{v_a}{\gamma} e^Z$
-------	-----------------------	--------------------------

T_c	T_b	$T_a \gamma^{\gamma-1}$
-------	-------	-------------------------

p_d		$\frac{p_a}{e^Z}$
-------	--	-------------------

v_d		$v_a e^Z$
-------	--	-----------

T_d		T_a
-------	--	-------

H_2	$T_a(\varphi_d - \varphi_a)$	$\frac{H_1}{\gamma^{\gamma-1}}$
-------	------------------------------	---------------------------------

W	$H_1 - H_2$	$H_1 \left(1 - \frac{1}{\gamma^{\gamma-1}} \right)$
-----	-------------	--

E	$1 - \frac{H_2}{H_1}$	$\left(1 - \frac{1}{\gamma^{\gamma-1}} \right)$
-----	-----------------------	--

$$\begin{aligned}
 R_\phi & \dots\dots\dots \frac{H_1}{T_b} \dots\dots\dots \frac{H_1}{T_a \gamma^{\gamma-1}} \\
 \text{M.E.T.} & \dots\dots\dots \frac{1}{2} \left(\frac{H_1 + H_2}{R_\phi} \right) \dots\dots\dots \frac{T_a}{2} (1 + \gamma^{\gamma-1}) \\
 R_v & \dots\dots\dots v_d - v_b \dots\dots\dots v_a \left(e^Z - \frac{1}{\gamma} \right) \\
 \text{M.E.P.} & \dots\dots\dots J \frac{W}{R_v} \dots\dots\dots J H_1 \left\{ \frac{1 - \frac{1}{\gamma^{\gamma-1}}}{v_a \left(e^Z - \frac{1}{\gamma} \right)} \right\} \\
 R_p & \dots\dots\dots p_d - p_b \dots\dots\dots p_a \left(\gamma^\gamma - \frac{1}{e^Z} \right) \\
 \text{M.E.V.} & \dots\dots\dots J \frac{W}{R_p} \dots\dots\dots J H_1 \left\{ \frac{1 - \frac{1}{\gamma^{\gamma-1}}}{p_a \left(\gamma^\gamma - \frac{1}{e^Z} \right)} \right\} \\
 R_T & \dots\dots\dots T_b - T_a \dots\dots\dots T_a (\gamma^{\gamma-1} - 1)
 \end{aligned}$$

CYCLE V.

Let Figs. 33 and 34 be the diagrams of the cycles.

CYCLE V.

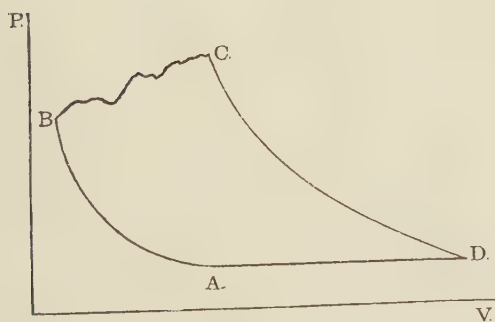


FIG. 33.

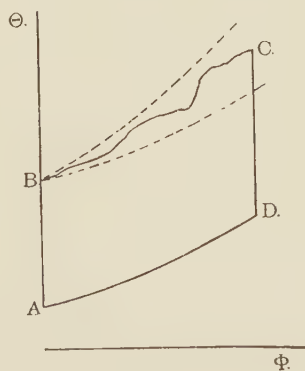


FIG. 34.

If we add heat at increasing p , v , and T the curves of states will lie somewhere between the isometric and isopiestic on both diagrams and the cycle is somewhere between III. and II. If the

heat addition took place at decreasing p , increasing v and j' the curve of states would lie between the isopiestic and the isothermal and the cycle lie between III. and IV. We cannot, however, calculate the appropriate set of formulæ without knowing the law of variation of states. The number of ways of variation is infinite, and while any one might be assumed, nothing could be gained by the calculation unless the law of variation chosen were preëminently simple or maintains in practice. Whatever it may be, however, the previous discussion will enable us to class it pretty well without entering much into details.

CYCLE VI.

Let Figs. 35 and 36 be the diagrams of the cycle.

CYCLE VI.

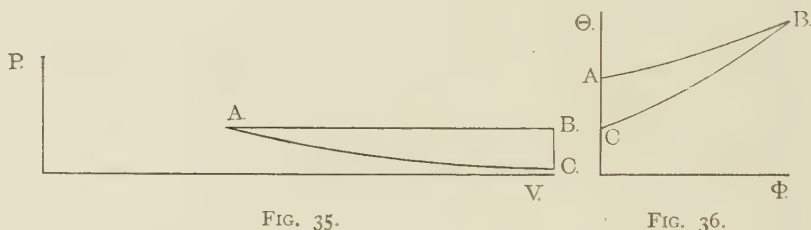


FIG. 35.

FIG. 36.

Heat being added isopiastically

$$T_b - T_a = \frac{H_1}{C_p}$$

$$\therefore T_b = T_a \left(1 + \frac{H_1}{C_p T_a} \right) = T_a x. \quad (1)$$

$$v_b = v_a \frac{T_b}{T_a} = v_a x \quad (2)$$

$$v_c = v_b = v_a x \quad (3)$$

The point C lies on the adiabat through A , hence

$$p_c = p_a \left(\frac{v_a}{v_c} \right)^\gamma = p_a \left(\frac{v_a}{v_a x} \right)^\gamma = \frac{p_a}{x^\gamma} \quad (4)$$

$$T_c = T_a \left(\frac{v_a}{v_c} \right)^{\gamma-1}$$

$$\therefore T_c = \frac{T_a}{x^{\gamma-1}}. \quad (5)$$

The perfect gas law is seen by inspection to apply

$$H_2 = C_v(T_b - T_c) = C_v \left(T_a x - \frac{T_a}{x^{\gamma-1}} \right)$$

$$\therefore H_2 = C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right) \quad (6)$$

$$* \quad W = H_1 - C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right) \quad (7)$$

$$E = 1 - \frac{C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right)}{H_1} \quad (8)$$

$$R\phi = \phi_b - \phi_a = C_p \log_e \frac{T_b}{T_a}$$

$$\therefore R\phi = C_p \log_e x \quad (9)$$

$$\text{M.E.T.} = \frac{1}{2} \left\{ \frac{1 + C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right)}{C_p \log_e x} \right\} \quad (10)$$

$$R_v = v'_c - v'_a = v_a(x - 1) \quad (12)$$

$$\text{M.E.P.} = J \frac{H_1 - C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right)}{v_a(x - 1)} \quad (13)$$

$$R_p = p_a - p_c = p_a \left(1 - \frac{1}{x^{\gamma}} \right) \quad (14)$$

$$\text{M.E.V.} = J \frac{H_1 - C_v T_a x \left(1 - \frac{1}{x^{\gamma}} \right)}{p_a \left(1 - \frac{1}{x^{\gamma}} \right)} = J \left\{ \frac{H_1}{p_a \left(1 - \frac{1}{x^{\gamma}} \right)} - \frac{C_v T_a x}{p_a} \right\} \quad (15)$$

$$R\phi = T_b - T_a = T_a(x - 1). \quad (16)$$

CYCLE VII.

Let Figs. 37 and 38 be its diagrams.

CYCLE VII.

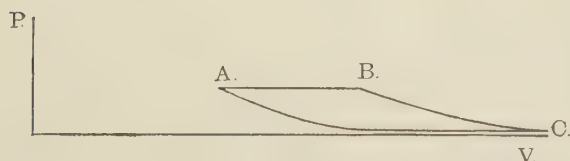


FIG. 37.

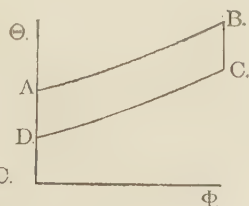


FIG. 38.

For B as before we have

$$v_b = v_a x, \quad (1)$$

$$p_b = p_a, \quad (2)$$

$$T_b = T_a x. \quad (3)$$

The point C lies on an adiabatic through B and is subject to the condition

$$p_a > p_c > 0 \quad (4)$$

$$v_c = v_b \left(\frac{p_b}{p_c} \right)^{\frac{1}{\gamma}} = v_a x \left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} \quad (5)$$

$$T_c = T_b \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} = T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \quad (6)$$

$$v_d = v_a \left(\frac{p_d}{p_a} \right)^{\frac{1}{\gamma}} = v_a \left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} = \frac{v_c}{x}. \quad (7)$$

But

$$v_b = v_a x.$$

Hence

$$\frac{v_a}{v_d} = \frac{v_b}{v_c}. \quad (8)$$

Similarly

$$\frac{T_a}{T_d} = \frac{T_b}{T_c}. \quad (9)$$

And

$$T_d = \frac{T_c}{x}$$

$$H_2 = C_p(T_c - T_d) = C_p T_c \left(1 - \frac{1}{x} \right) \quad (10)$$

$$= C_p T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{x} \right)$$

$$W = H_1 - H_2 = H_1 - C_p T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{x} \right) \quad (11)$$

$$E = 1 - \frac{H_2}{H_1} \quad (12)$$

$$R\varphi = C_p \log_e x \text{ as in VI.} \quad (13)$$

$$\therefore \text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{C_p \log_e x} \right) \quad (14)$$

$$R_v = v_c - v_a = v_a \left[\left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} x - 1 \right] \quad (15)$$

$$\text{M.E.P.} = J \frac{W}{R_v} = J \frac{W}{v_a \left[\left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} x - 1 \right]} \quad (16)$$

$$R_p = p_a - p_c = p_a - p_d \quad (17)$$

$$\text{M.E.V.} = J \frac{W}{p_a - p_d} \quad (18)$$

$$R_T = T_b - T_a = T_a(x - 1) \text{ as in VI.} \quad (19)$$

CYCLE VIII.

Figs. 39 and 40 are its diagrams.

CYCLE VIII.

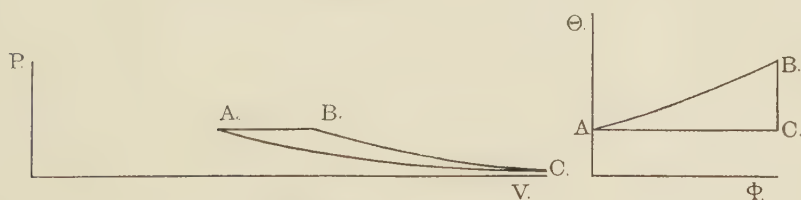


FIG. 39.

FIG. 40.

We have for B

$$v_b = v_a x, \quad (1)$$

$$p_b = p_a, \quad (2)$$

$$T_b = T_a x. \quad (3)$$

The isothermal through A intersects the adiabatic through B to determine C .

From the adiabatic

$$p_c = p_b \left(\frac{v_b}{v_c} \right)^{\gamma}.$$

From the isothermal

$$p_c = p_a \frac{v_a}{v_c}$$

$$\therefore p_b \left(\frac{v_b}{v_c} \right)^{\gamma} = p_a \frac{v_a}{v_c}.$$

But

$$v_b^{\gamma} = v_a^{\gamma} x^{\gamma}$$

$$\therefore \frac{v_a^\gamma x^\gamma}{v_a} = \frac{v_c^\gamma}{v_c}$$

By substitution

$$\therefore v_c = v_a x^{\frac{\gamma}{\gamma-1}}. \quad (4)$$

$$p_c = \frac{p_a}{x^{\frac{\gamma}{\gamma-1}}}$$

$$H_2 = T_a(\phi_b - \phi_a) = T_a C_p \log_e x \quad (6)$$

$$W = H_1 - T_a C_p \log_e x \quad (7)$$

$$E = 1 - \frac{T_a C_p \log_e x}{H_1} \quad (8)$$

$$R_v = v_c - v_a = v_a(x^{\frac{\gamma}{\gamma-1}} - 1) \quad (9)$$

$$\text{M.E.P.} = \frac{H_1 - T_a C_p \log_e x}{v_a(x^{\frac{\gamma}{\gamma-1}} - 1)} \quad (10)$$

$$R_p = p_a - p_c = p_a \left(1 - \frac{1}{x^{\frac{\gamma}{\gamma-1}}} \right) \quad (11)$$

$$\text{M.E.V.} = \frac{H_1 - T_a C_p \log_e x}{v_a(x^{\frac{\gamma}{\gamma-1}} - 1)} \quad (12)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + T_a C_p \log_e x}{C_p \log_e x} \right) = \frac{1}{2} \left(\frac{H_1}{C_p \log_e x} + T_a \right) \quad (13)$$

CYCLE IX.

Let Figs. 41 and 42 be its diagrams.

CYCLE II.

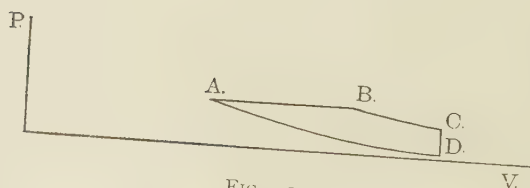


FIG. 41.

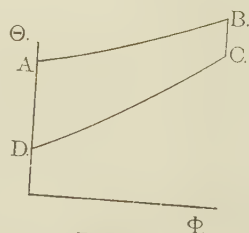


FIG. 42.

Up to the point C the results of VII. may be assumed.

The point D lies on an adiabat through A and is subject to the conditions

$$v_d = v_c \quad (1)$$

$$p_c > p_d > 0 \quad (2)$$

$$v_d > v_b \quad (3)$$

$$p_d = p_a \left(\frac{v_a}{v_d} \right)^\gamma = p_a \left(\frac{v_a}{v_c} \right)^\gamma = p_a \left\{ \frac{v_a}{v_a x \left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}}} \right\}^\gamma = \frac{p_a p_c}{p_a x^\gamma} = \frac{p_c}{x^\gamma} \quad (4)$$

$$T_d = T_c \frac{p_d}{p_c} = \frac{T_c}{x^\gamma} \quad (5)$$

$$\begin{aligned} H_2 &= C_v(T_c - T_d) = C_v T_c \left(1 - \frac{1}{x^\gamma} \right) \\ &= C_v T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{x^\gamma} \right) \end{aligned} \quad (6)$$

$$W = H_1 - H_2 = H_1 - C_v T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{x^\gamma} \right) \quad (7)$$

$$E = 1 - \frac{H_2}{H_1} = 1 - \frac{C_v T_a x \left(\frac{p_c}{p_a} \right)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{1}{x^\gamma} \right)}{H_1} \quad (8)$$

$$R_\phi = C_p \log_e x \quad \text{as before} \quad (9)$$

$$\text{M.E.T.} = \frac{1}{2} \left(\frac{H_1 + H_2}{C_p \log_e x} \right) \quad (10)$$

$$R_v = v_c - v_a = v_a \left[x \left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} - 1 \right] \quad (11)$$

$$\text{M.E.P.} = J \frac{W}{v_a \left[x \left(\frac{p_a}{p_c} \right)^{\frac{1}{\gamma}} - 1 \right]} \quad (12)$$

$$R_p = p_a - p_d = p_a - \frac{p_c}{x^\gamma} \quad (13)$$

$$\text{M.E.V.} = J \frac{W}{p_a - \frac{p_c}{x^\gamma}} \quad (14)$$

$$R_T = T_b - T_a = T_a(x - 1). \quad (15)$$

CYCLE X.

In this cycle as in the last four heat is added at atmospheric pressure, then follows adiabatic expansion after which heat is abstracted according to some law as yet undefined. Adiabatic compression completes the cycle. As the law of abstraction of heat is as yet undefined we cannot, of course, derive formulæ for the cycle and will leave its discussion as we did Cycle V.

We might have derived formulæ for the imperfect carrying out of cycles VI., VII., VIII. and IX. but they are of such slight importance in practice that it did not seem desirable.

Besides the twenty-two cycles considered there may be others due to the combination or differentiation of these typical ones, but the object of this paper will be best accomplished by a study of types, the non-typical or synthetic cycles having been omitted. The method of study here set forth being of universal application to all possible cycles will furnish means of reaching a clear understanding of any of the unconsidered cycles should need arise.

COMPARISON OF CYCLES.

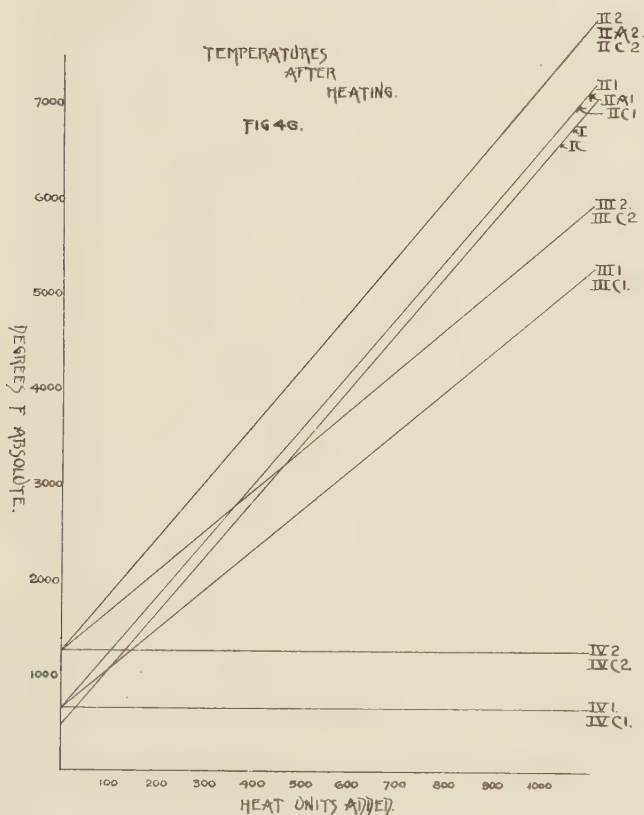
Of the many cycles considered we will choose for comparison only those that might be called the perfect cycles because accurately defined and these are Cycles I., I. C, II., II. A₂, II. C, III., III. C, IV., IV. C. The atmospheric cycles are of comparatively little importance and will be neglected in the comparison. We will take up each variable separately and study its value in the different cases by formula and by calculated examples expressed in curves which are then the graphical formulæ. The curves given are approximately correct and as the same approximation will probably maintain for all the cases the curves will serve as well for comparison as if absolutely exact. Two cases of each are given, one with compression 2 : 1 and one with 10 : 1 (volume ratios). Call the atmospheric values p_a , v_a , T_a .

TEMPERATURES AFTER ADDITION OF H_1 , B. T. U.

Cycle.

$$\text{I., I. C} \quad T_c = T_a X = T_a \left(1 + \frac{H_1}{C_v T_a} \right) \quad (1)$$

$$\text{II., II. A., II. C} \quad T_c = T_b X = T_b \left(1 + \frac{H_1}{C_v T_b} \right) \quad (2)$$



$$\text{III., III. C} \quad T_c = T_b X = T_b \left(1 + \frac{H_1}{C_v T_b} \right) \quad (3)$$

$$\text{IV., IV. C} \quad T_c = T_b. \quad (4)$$

Using axes of T_c and H_1 we see these are all straight lines passing through the axis of temperatures at T_b above the origin except in cycles (I., I. C) where the intersection is at T_a . These lines are inclined to the axis of H and make with it an angle α such that in

$$\text{I., I. C, II., II. A, II. C} \quad \tan \alpha = \frac{1}{C_v}. \quad (5)$$

and in

$$\text{III., III. C} \quad \tan \alpha' = \frac{I}{C_p} \quad (6)$$

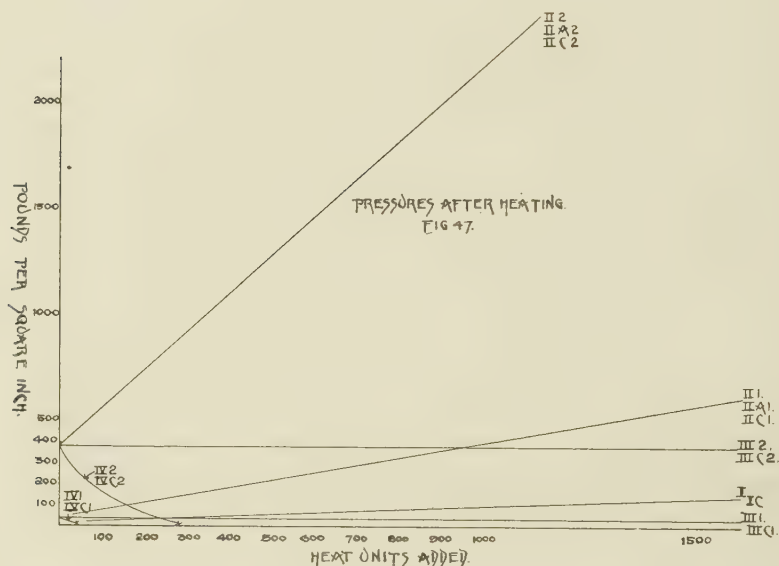
while IV., IV. C are lines parallel to axis H_1 .

These lines are shown in Fig. 46 for two compressions.

PRESSURES AFTER ADDITION OF H_1 , B. T. U.

Cycle.

$$\text{I., I. C} \quad p_c = p_a X = p_a \left(1 + \frac{H_1}{C_p T_b} \right) \quad (7)$$



$$\text{II., II. A, II. C} \quad p_c = p_b X = p_b \left(1 + \frac{H_1}{C_r T_b} \right) \quad (8)$$

$$\text{III., III. C} \quad p_c = p_b \quad (9)$$

$$\text{IV., IV. C} \quad p_c = \frac{p_b}{e^Z} = \frac{p_b}{e^{(C_p - C_v) T_b / H_1}} \quad (10)$$

Equations (7), (8) and (9) are all straight lines, (9) being parallel to axis H_1 while (7) and (8) are inclined. Equation (10) is an exponential curve sloping down to the right and concave up and asymptotic to axis of H as can be seen from the derivatives

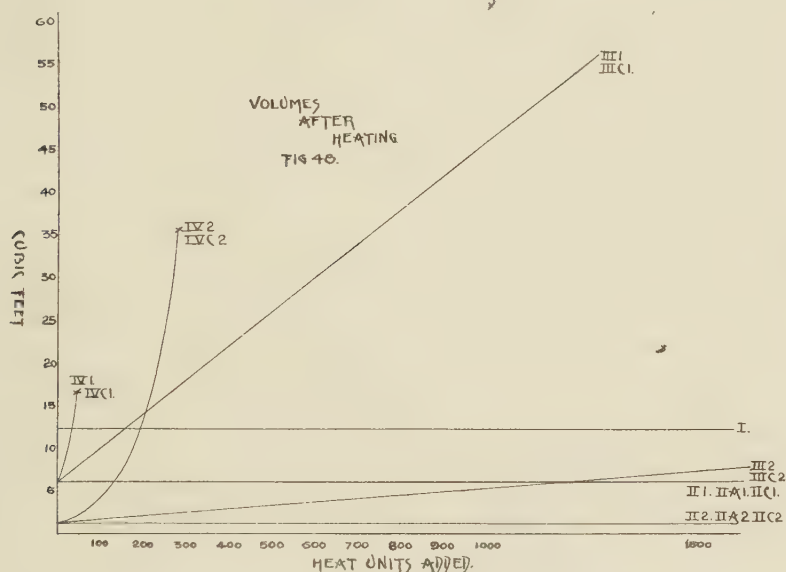
$$\frac{dp}{dH_1} = - \frac{p_b}{(C_p - C_v) T_b e^{\frac{H_1}{(C_p - C_v) T_b}}}, \quad (11)$$

$$\frac{d^2 p}{dH_1^2} = \frac{p_b}{(C_p - C_v)^2 T_b^2 e^{\frac{H_1}{(C_p - C_v) T_b}}}. \quad (12)$$

These formulæ are given in Fig. 47 for the two cases.

VOLUMES AFTER HEATING BY H_1 , B. T. U.

Cycle.



$$\text{I., I. C} \quad v_c = v_a \quad (13)$$

$$\text{II., II. A, II. C} \quad v_c = v_b \quad (14)$$

$$\text{III., III. C} \quad v_c = v_b Y = v_b \left(1 + \frac{H_1}{C_v T_b} \right) \quad (15)$$

$$\text{IV., IV. C} \quad v_c = v_b e^Z = v_b e^{\frac{H_1}{(C_p - C_v) T_b}}. \quad (16)$$

Formula (13) is a straight line parallel to H_1 and is always less than (14) which is similar but cuts axis of V_c at a point v_b higher than v_a . Equation (15) is a straight line inclined to H_1 . Equation (16) is an exponential curve cutting axis V_c at point V_b , it is concave up and slopes up to the right as is shown by the derivatives

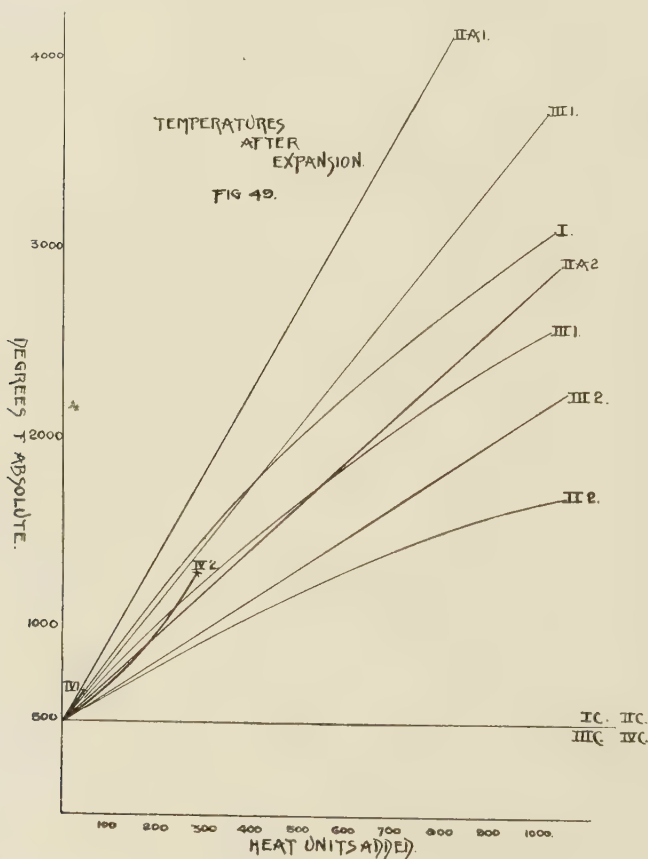
$$\frac{dv_c}{dH_1} = \frac{v_b}{(C_p - C_v)} T_b e^{\frac{H_1}{(C_p - C_v) T_b}}, \quad (17)$$

$$\frac{d^2 v_c}{dH_1} = \frac{v_b}{(C_p - C_v)^2} T_b^2 e^{\frac{H_1}{(C_p - C_v) T_b}}. \quad (18)$$

These curves are shown in Fig. 48 for the two cases.

TEMPERATURE AFTER EXPANSION.

Cycle.



$$\text{I.} \quad T_d = T_a(X)^{\frac{1}{\gamma}} = T_b \left(1 + \frac{H_1}{C_v T_a} \right)^{\frac{1}{\gamma}}, \quad (19)$$

$$\text{I. C.} \quad T_d = T_a, \quad (20)$$

$$\text{II.} \quad T_d = T_a(X)^{\frac{1}{\gamma}} = T_a \left(1 + \frac{H_1}{C_v T_b} \right)^{\frac{1}{\gamma}}, \quad (21)$$

$$\text{II. A} \quad T_d = T_a X = T_a \left(1 + \frac{H_1}{C_v T_b} \right), \quad (22)$$

$$\text{II. C} \quad T_d = T_a, \quad (23)$$

$$\text{III.} \quad T_d = T_a Y = T_a \left(1 + \frac{H_1}{C_p T_b} \right), \quad (24)$$

$$\text{III. C} \quad T_d = T_a, \quad (25)$$

$$\text{IV.} \quad T_d = T_a e^{Y-1} = T_a e^{\frac{H_1}{C_p T_b}}, \quad (26)$$

$$\text{IV. C} \quad T_d = T_a, \quad (27)$$

Curves (19) and (21) are similar in form, cutting axis T_d at different points, however, and having different slopes. It is easily seen that (21) is always greater than (19), also that (22) is greater than (21) since

$$\frac{1}{\gamma} < 1.$$

Both (22) and (24) are straight lines, but they have different slopes through intersecting axis T_d at same point

$$(\tan \delta)_{\text{II. A.}} = \frac{T_a}{C_v T_b} = \frac{1}{\gamma^{\gamma-1} C_v}, \quad (28)$$

$$(\tan \delta)_{\text{III.}} = \frac{1}{\gamma^{\gamma-1} C_p}. \quad (29)$$

whence (22) is always greater than (24). Equation (26) is an exponential cutting T_d axis at T_a , it is concave up and slopes up to the right since

$$\frac{dT_d}{dH_1} = \frac{1}{\gamma^{\gamma-1} C_p} e^{\frac{H_1}{C_p T_b}}, \quad (30)$$

$$\frac{d^2 T_d}{dH_1^2} = \frac{1}{\gamma^{\gamma-1} C_p^2 T_b} e^{\frac{H_1}{C_p T_b}}. \quad (31)$$

These curves are shown in Fig. 49 for the two cases.

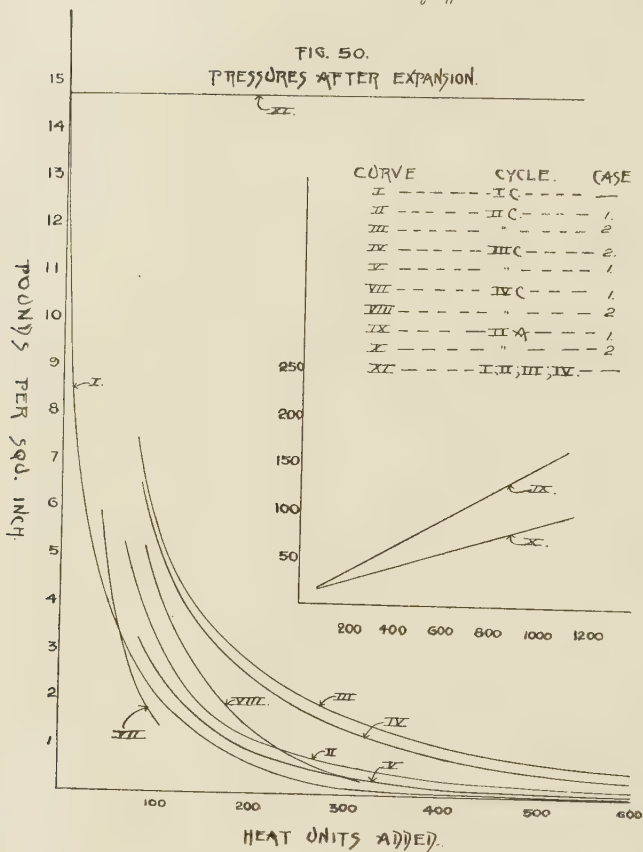
PRESSURES AFTER EXPANSION.

Cycle.

$$\text{I.} \quad p_d = p_a. \quad (32)$$

I. C

$$p_d = \frac{p_a}{X^{\frac{1}{\gamma-1}}} = \frac{p_a}{\left(1 + \frac{H_1}{C_v T_a}\right)^{\frac{1}{\gamma-1}}}, \quad (33)$$



II.

$$p_d = p_a, \quad (34)$$

II. A

$$p_d = p_a X = p_a \left(1 + \frac{H_1}{C_v T_b}\right), \quad (35)$$

II. C

$$p_d = \frac{p_a}{X^{\frac{1}{\gamma-1}}} = \frac{p_a}{\left(1 + \frac{H_1}{C_v T_b}\right)^{\frac{1}{\gamma-1}}}, \quad (36)$$

III.

$$p_d = p_a, \quad (37)$$

III. C

$$p_d = \frac{p_a}{X^{\frac{1}{\gamma-1}}} = \frac{p_a}{\left(1 + \frac{H_1}{C_p T_b}\right)^{\frac{1}{\gamma-1}}}, \quad (38)$$

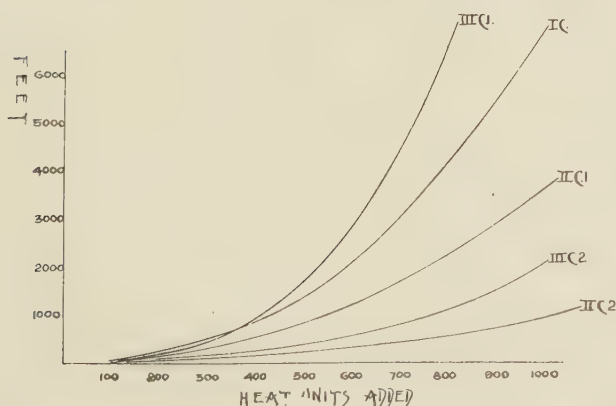
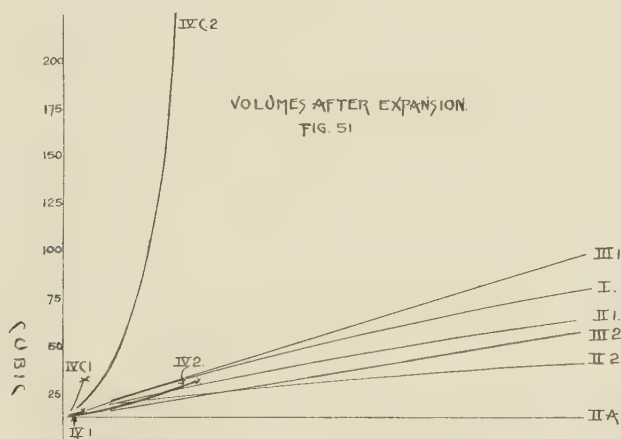
$$\text{IV.} \quad p_d = p_a, \quad (39)$$

$$\text{IV. C} \quad p_d = \frac{p_a}{e^Z} = \frac{p_a}{\frac{H_1}{e^{(C_p - C_v)T_b}}}. \quad (40)$$

Equations (32), (34), (37), (39) are identical and represent a straight line parallel to axis H_1 . Curve (55) is a straight line inclined to H_1 . All the others are concave up sloping down to the right; their relative positions are seen in Fig. 50 for two compressions.

VOLUMES AFTER EXPANSION.

Cycle.



$$\text{I.} \quad v_d = v_a X^{\frac{1}{\gamma}} = v_a \left(1 + \frac{H_1}{C_v T_a} \right)^{\frac{1}{\gamma}}, \quad (41)$$

$$\text{I. C} \quad v_d = v_a X^{\frac{1}{\gamma-1}} = v_a \left(1 + \frac{H_1}{C_v T_a} \right)^{\frac{1}{\gamma-1}}, \quad (42)$$

$$\text{II.} \quad v_d = v_a X^{\frac{1}{\gamma}} = v_a \left(1 + \frac{H_1}{C_v T_b} \right)^{\frac{1}{\gamma}}, \quad (43)$$

$$\text{II. A} \quad v_d = v_a, \quad (44)$$

$$\text{II. C} \quad v_d = v_a X^{\frac{1}{\gamma-1}} = v_a \left(1 + \frac{H_1}{C_v T_b} \right)^{\frac{1}{\gamma-1}}, \quad (45)$$

$$\text{III.} \quad v_d = v_a Y, \quad (46)$$

$$\text{III. C} \quad v_d = v_a Y^{\frac{1}{\gamma-1}} = v_a \left(1 + \frac{H_1}{C_p T_b} \right)^{\frac{\gamma}{\gamma-1}}, \quad (47)$$

$$\text{IV.} \quad v_d = v_a e^{Y-1} = v_a e^{\frac{H_1}{C_p T_b}}, \quad (48)$$

$$\text{IV. C} \quad v_d = v_a e^Z = v_a e^{\left(\frac{H_1}{C_p T_b} - \frac{H_1}{C_v T_b} \right)}, \quad (49)$$

These curves will admit of considerable discussion, but the curves of Fig. 51 show at a glance all we wish to know in general.

HEAT DISCHARGED OR ABSTRACTED.

Cycle.

$$\text{I.} \quad H_2 = C_p T_a (X^{\frac{1}{\gamma}} - 1), \quad (50)$$

$$\text{I. C} \quad H_2 = T_a C_e \log_e X = T_a C_v \log_e \left(1 + \frac{H_1}{C_v T_b} \right), \quad (51)$$

$$\text{II.} \quad H_2 = C_p T_a (X^{\frac{1}{\gamma}} - 1), \quad (52)$$

$$\text{II. A} \quad H_2 = C_v T_a (X - 1) = \frac{H_1}{\gamma - 1}, \quad (53)$$

$$\text{II. C} \quad H_2 = C_v T_a \log_e \left(1 + \frac{H_1}{C_v T_b} \right), \quad (54)$$

$$\text{III.} \quad H_2 = C_p T_a (Y - 1) = \frac{H_1}{\gamma - 1}, \quad (55)$$

$$\text{III. C} \quad H_2 = C_p T_a \log_e \left(1 + \frac{H_1}{C_p T_b} \right), \quad (56)$$

$$\text{IV.} \quad H_2 = C_p T_a (e^{Y-1} - 1) = C_p T_a (e^{\frac{H_1}{C_p T_b}} - 1), \quad (57)$$

IV. C

$$H_2 = \frac{H_1}{\gamma^{\gamma-1}}. \quad (58)$$

Equations (33), (55) and (58) are identical, that is, these three cycles will discharge the same amount of heat and have the same efficiency; moreover this efficiency will be independent of everything but the compression. These three cycles have, further, a common property not seen by the formula, but from their definitions each receives and discharges all its heat according to the same law.

Cycle II. A receives all heat at constant volume and discharges all at constant volume.

Cycle III. receives all heat at constant pressure and discharges all at constant pressure.

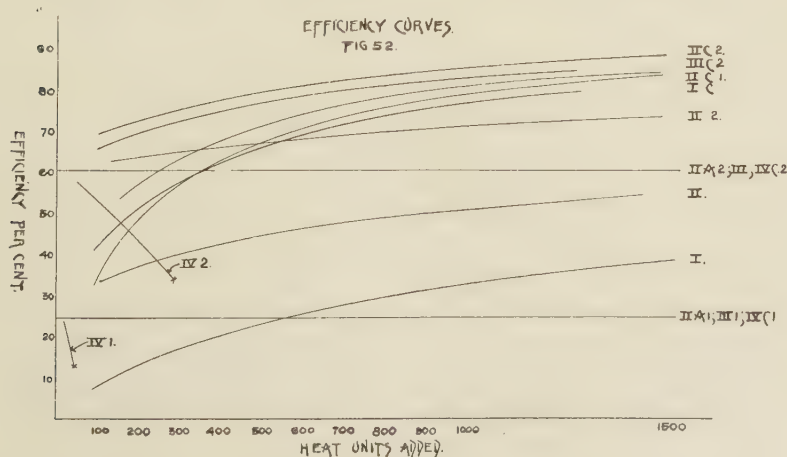
Cycle IV. C receives all heat at constant temperature and discharges all at constant temperature.

A consideration of the above would seem to warrant the proposition :

When all the heat is discharged according to the same law under which it was received then the cycle will have an efficiency independent of everything but the previous compression and will be given by

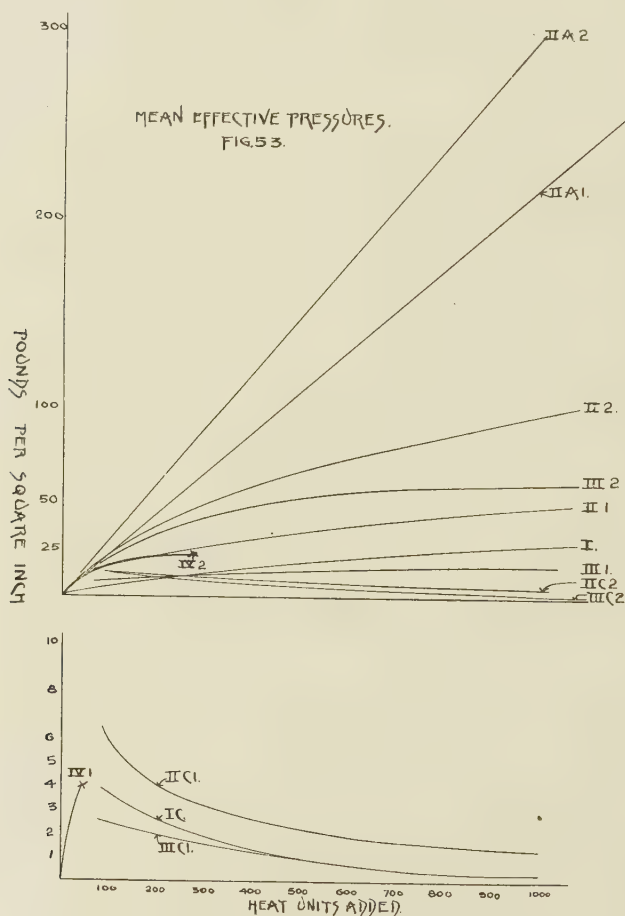
$$E = 1 - \frac{1}{\gamma^{\gamma-1}}.$$

We may remark here that as IV. C is the Carnot Cycle we can state that Cycles II. A and III. have the same efficiency as the



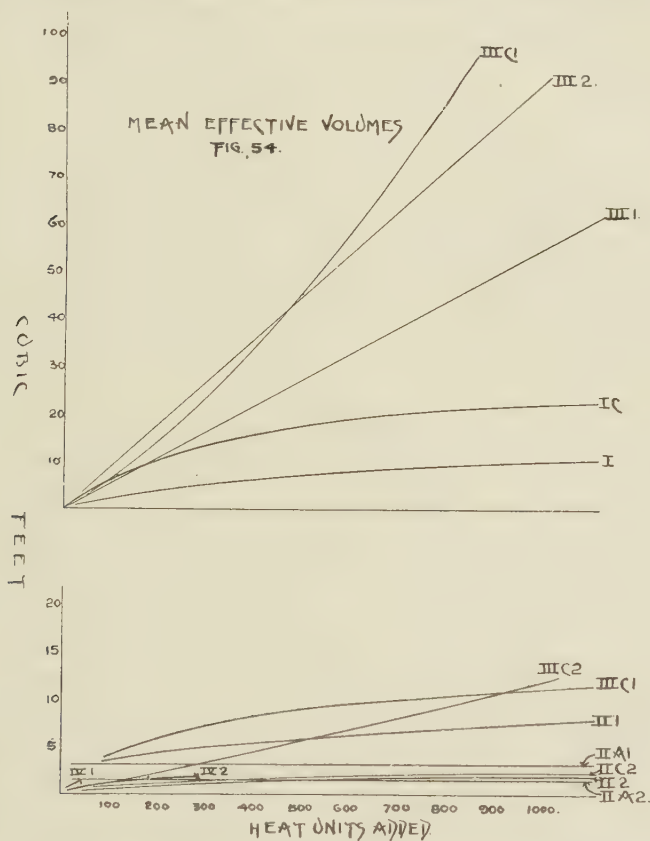
Carnot Cycle with same previous compression. This is an important supplementary to the old theorem that the Carnot Cycle has the highest efficiency for its temperature range.

The relation between the other values of H_2 are best shown by the curves of Fig. 52 by implication. The quantities: Pressure range, Volume range, Temperature range, do not need separate



rate sets of curves as we can get a fair idea of the values from an inspection of the previous curves. If, however, any case seems to call for an exact solution it can be obtained by a simple substitution in the formulæ already given.

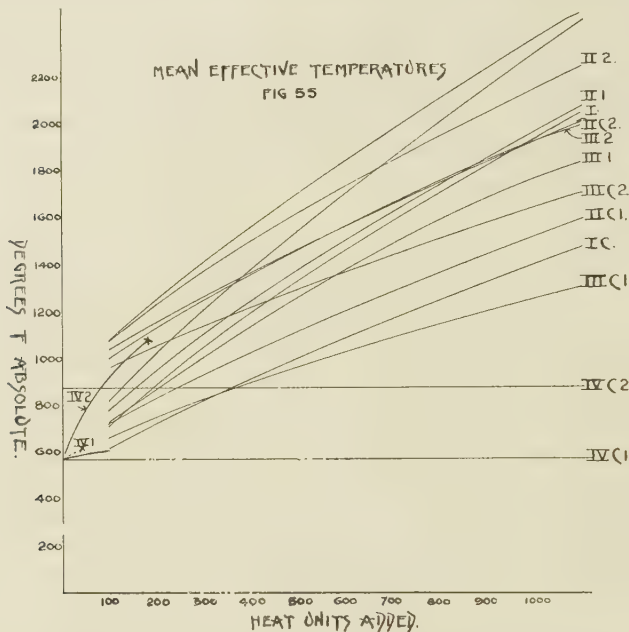
Mean effective pressure, volume and temperature, however, are important values and not easily located relatively from the formulæ. Figs. 53, 54 and 55 show these curves as calculated for two cases of compression. It may be here remarked that in the case of Cycle IV, when the compression is 2:1 only about 44 B.T.U. can be added to 1 lb. air and with a compression of 10:1 about 282 B.T.U., this is why the curves end abruptly at these values of H_1 .



A thorough discussion of the equations derived while important and leading no doubt to many new and useful results would be very long and would extend beyond the limits set for this paper which had for its object rather the exposition of the method of procedure than a thorough application of that method.

Besides the complete discussion referred to there is another important point of view to be taken of these formulæ—that of inter-

pretation with respect to operating engines ; this is also reserved for later treatment.



The curves of the important cyclic variables as functions of the heat supplied admit in their interpretation of the statement of many important new propositions. Some of these are quite general, while others are more specific. A few of the most obvious will be noted.

The cycle consists of a series of operation or pressure volume temperature changes resulting in a return to the original state of pressure volume and temperature.

GENERAL PROPOSITIONS.

1. The P.V.T. at any point of a cycle depends on: (a) The cycle itself qualitatively considered, *i. e.*, the nature and order of succession of the processes or phases already completed; (b) the extent or intensity of each phase of the cycle qualitatively; (c) the amount of heat, H , added before reaching the point considered. For example, the temperature at the end of combustion will be different for different cycles, will vary with the compression before heating, the law of compression and the amount of heat added.

2. The part of the total heat transformed into work is a function of the cycle, and will vary with the order, nature and extent of the cyclic phases, except when all the heat is added and all abstracted according to the same law.

3. When the laws of heating and of cooling are identical then the part of the total heat supplied that becomes transformed into work is constant for the same previous compression, and this resulting efficiency is a function of the previous compression only when these other two phases, compression and expansion, completing the cycle, have likewise the same law.

4. The range of changes in pressure volume and temperature is different for different cycles, and in any one cycle will depend on the amount of heat added.

5. While the variations noted do in general hold, yet in the different cycles each variable may be a different function of H_1 , so that two or more curves may intersect, and for that particular value of H_1 the variable will have the same value in two or more different cycles simultaneously.

EFFICIENCIES (Fig. 52).

By inspection and plotting of formulæ Nos. 40-58, page 90, for the values of heat necessarily discharged we may draw some conclusions concerning efficiencies of the transformation process for various cycles. Denoting as before the ratio of heat energy transformed to that supplied by E = efficiency, it will be possible to draw the following comparisons:

6. For Cycles II. A_2 , III., IV. C the efficiency is a function of the adiabatic compression only and the same function for each. It is independent of the amount of heat supplied, *i. e.*, is not a function of H .

7. For all cycles the efficiency increases with the compression, but not according to the same law.

8. For Cycles IV., IV. A, IV. B, IV. C the efficiency decreases with increase of heat added to the same mass of gas.

9. For all other cycles except II. A_2 , III., IV. C the efficiency increases with H_1 , but according to different laws, so that the distance between efficiency curves will vary.

10. For these cases a change in H_1 will produce more effect when H_1 is small than when it is large.

11. After heat has been added the efficiency will vary with the

degree of expansion. Cycle II., therefore, will have an efficiency always higher than II. A and lower than II. B or II. C.

12. Cycles in which an adiabatic compression precedes heating, will always have a higher efficiency than those lacking this compression, other things being equal.

13. For the same initial conditions and same heat added if H_1 is large enough Cycle II. C will always have the highest efficiency

and then come in order III. C; I. C; II. $\left. \begin{array}{l} \text{II. A} \\ \text{III.} \\ \text{IV. C} \end{array} \right\}$ always re-

membering that IV., IV. A, IV. B, IV. C cannot exist if H_1 be large.

14. The difference in efficiency between the curtailed expansion of Cycle II. A₂ and that of II. increases with the amount of heat, the difference being small when H_1 is small and greater as H_1 increases, the greatest possible being about 12 per cent.

15. Expanding Cycle II. to original temperature making Cycle II. C, may increase the efficiency from 5 to 15 per cent. approximately for possible values of H_1 .

16. Cycle III. may add by expansion to original temperature as much as 25 per cent. to the efficiency for possible values of H_1 .

17. Cycles IV., IV. A, IV. B have an efficiency decreasing with increase of H provided H remain small; when H passes a certain limit the cycle ceases to be possible.

18. A change in the volume ratio of compression from $\frac{1}{2}$ to $\frac{1}{10}$ will increase the efficiency of the cycles as follows for possible values of H_1 .

Cycle II.	. . .	30-20 per cent. approximately, depending on H_1 ,
" II. A ₂	}	. . . 35 per cent. approximately, depending on H_1 ,
" III.		
" IV. C		
" II. C	. . .	40-5 per cent. approximately, depending on H_1 .

TEMPERATURES (Figs. 46, 49, 55).

1. For the same previous compression the temperature resulting in each cycle from heat addition and which is the maximum for the cycle, will be different. That is, the addition of the same amount of heat will result in a different temperature for each group of cycles and the lines of Fig. 46 show that no two can be identical except I. and III.; which cross.

2. Gases passing through Cycle I. may, on addition of a certain amount of heat, H_1 , have a temperature equal to what the same gas would have passing through Cycle III. However, for more heat added the temperature for I. will become higher than that for III. while for less heat added III. will be higher.

3. Increase of compression before heating changes the temperature after heating by only so much numerically as the varied compression has resulted in changing the temperature before heating begins.

4. The temperature increase due to heating is proportional to the amount of heat added H_1 , and the constant of proportionality involves the reciprocal of the specific heat for the process and the weight of the gas present.

5. After the gas has expanded to the greatest volume possible in the cycle no two cycles will leave the gas with the same temperature except in a few special cases.

6. Cycle I. C, II. C, III. C, IV. C by definition have the same temperatures at the end of expansion, and this is moreover constant no matter what H may be and is equal to the initial temperature of the cycle.

7. There will be a value of H_1 for a limited range of compressions for which cycle III. may give to the gas the same final expansion temperature as Cycle I.

8. Similarly II. for one compression may coincide in final temperature with II. A for some other compression.

9. The temperature after expansion for cycle II. A, will always be higher than for III. and III. higher than for II.

10. In round numbers II. A may be 25 per cent. higher than III. and may even be 100 per cent. higher than II. for the same compression for possible values of H_1 .

11. With variation of compression the temperature at the termination of expansion will vary, always becoming lower but the extent of the lowering will depend on how much heat was added before expansion and in case II. A and III. is exactly proportional to H_1 .

12. A change of compression $\frac{1}{2}$ to $\frac{1}{10}$ may change the temperature at the end of expansion in the case of cycle II. A and III. as much as 80 per cent. for possible values of H_1 .

13. Mean effective temperature, Fig. 55, are different for different cycles and for different compressions in the same cycle.

14. Cycle IV. C is the only cycle with constant mean effective temperature.

15. Mean effective temperature of all other cycles increase with H_1 .

16. For large values of H_1 the order of magnitude of mean effective temperatures will be: Lowest, IV. C, III. C, I. C, II. C, III., I., II., highest, II. A.

17. For lower values of H_1 this order may be somewhat changed and there will be points at which two different cycles will have simultaneous values of M.E.T. and H_1 .

PRESSURES (Figs. 47, 50, 53).

1. The pressures resulting from heat addition are different for cycles with different numerals, but the same in any one group. Thus, II., II. A, II. B, II. C or Group II. will all have the same pressures, whereas those of Group II. will differ from those of Groups III. and IV.

2. Lines representing pressures or functions of the heat supplied, H_1 , will cross as these functions are different for different groups, and it will hence be possible for the different groups of cycles to have the same pressures for certain values of H_1 .

3. Groups I. and II. have pressures after heating that increase with H_1 , while in Group III. the pressure is constant and in IV. decreasing with increase of H_1 .

4. For same compressions Group II. will always have the highest pressure after heating, and III., IV. and I. come in the order named for moderate H_1 , while for large H_1 IV. cannot exist.

5. Increase of compression will change the pressure after heating in Group III. only so much as results from the changed compression before heating. In Groups II. and I. the change is such as to keep the pressure ratio before and after heating constant; so that for a given change in H_1 the resulting pressure change in II. will be greatest for high compressions, less for moderate compressions and least for no compression, *i. e.*, for Group I.

6. After expansion by definition the pressures of I., II., III. and IV. are all atmospheric and equal.

7. The pressure which II. A₂ will reach when the gas has expanded to original volume increases with H_1 and is such that the ratio of this pressure to atmospheric is the same as the ratio of pressure after heating to that before.

8. Cycles with letter *C* all go below atmosphere in expanding to such a pressure as will bring the temperature down to that originally existing in the gas. These resulting pressures after expansion are different for each cycle, but the lines representing them as functions of H_1 may intersect.

9. The lines for IV. *C* may cross others, but I. *C*, II. *C*, III. *C* cannot intersect and these will always be in the order of magnitude II. *C*, III. *C*, I. *C* and all asymptotic to axis of H_1 so that the terminal pressure can never be zero.

10. An increase of compression will cause an increase in final pressure for same H_1 .

11. Mean effective pressure expressed as a function of H_1 will give for every cycle and every different compression a different M.E.P. curve, but as before these may intersect.

12. For all cycles except those ending with isothermal return to the original state, the M.E.P. increases with H_1 but for those bearing the letter *C* the M.E.P. decreases and for no cycle is it constant.

13. For the same previous compression the cycles have M.E.P. of about the following order of magnitude when H_1 is large enough.

Greatest M.E.P., II. A_2 , 200; II., 40; I., 25; III., 15; II. *C*, 1.5; I. *C*, 0.3; III. *C*, 0.2. When H_1 is small IV. will probably come between III. and II. *C*.

14. A change in compression from $\frac{1}{2}$ to $\frac{1}{10}$ (vols.) may cause a change in II. A_2 of 35 per cent., II of 100 per cent., III. of 300 per cent. for the same possible values of H_1 .

15. The effect of changed compression before heating is the more marked on M.E.P. resulting when M.E.P. is lowest and the extent of the increase is greater with H_1 .

VOLUMES (Figs. 48, 51, 54).

1. The volumes after heating are the same for cycles of the same group and for all groups increase with H_1 except in Groups I. and II. where by definition they are constant and equal to the volumes existing before heating.

2. In Group III. the volumes after heating are proportional to H_1 with the same constant of proportionality for the same compression. Increase of compression decreases this constant of proportionality.

3. In Group IV. the volumes increase rapidly with H_1 but are not proportional to H_1 so long as H_1 is small; with large H_1 Group IV. cannot exist.

4. Lines of volumes after heating represented as functions of H_1 may cross in some cases. II., IV. and III. may cross I., *i. e.*, the compression cycles may cross the non-compression ones. But for the same compression II., III. and IV. can never have the same volumes after heating. Lines of III. and IV. for high compression may cross II. for a lower compression but cannot cross each other.

5. For possible values of H_1 the volumes after heating for the different groups may have the following order of magnitude if H_1 is large enough: Group III., 55.00; group I., 12.38; group II., 6.00.

6. After expansion is completed the volume occupied by the gas in the different cycles will vary through very wide limits, increasing with H_1 .

7. The volume occupied by Cycle III. will be such as to keep the ratio between this final volume and the volume before compression the same as the ratio of volume after heating to that before and the final volume is proportional to H_1 . The constant of proportionality is decreased by compression increase.

8. The final volume of Cycle II. A is least and equal to that existing before compression.

9. When H_1 is large enough there may be a value for which the final volume may exist in the following order of magnitude: III. C₁, 7,000.00; I. C, 4,200.00; II. C, 2,300.00; III., 75.00; I., 65.00; II., 51.00; II. A₂, 12.38. A change of compression by which the volume after compression is one fifth that for the previous case may change this list to the following: III. C., 1,000.00; I. C, 4,200.00; II. C, 500.00; III., 40.00; I., 65.00; II., 34.00; II. A₂, 12.38.

10. The mean effective volumes increase with H_1 for all cycles except II. A₂ in which this variable is constant.

11. For cycle III. the M.E.V. is proportional to H_1 and increase of compression increases the constant of proportionality.

From the data here set down the selection of a cycle on purely ideal grounds can be made with a full knowledge of all the conditions surrounding the selection; that is knowing what results are

desired the cycle that theoretically, ideally or mathematically considered gives the results can be found and in addition it is easy to see what accompanying circumstances are inevitable. If that cycle that transforms the greatest amount of heat into work ideally is wanted it is readily seen that II. C with as high compression as possible must be selected, but it is also evident that a very large volume range must be submitted to. If that cycle with the lowest temperature range is wanted then any of Group IV. must be taken.

If a cycle is desired that will convert of any amount of heat the same proportion into work then any one of II. A, III. or IV. C, but of these one has the lowest pressure range, another the lowest temperature range and the last the lowest volume range.

Examples could be multiplied almost indefinitely, but enough has been said to make clear the purpose of and justify this laborious analysis, for the results desired can be set down at once for cycles considered, and, moreover, for any cycle not considered it is evident that similar treatment will place it at once in comparison with all these presented.

NOTE.—This paper is sent to you that you may examine it in advance of the meeting, and prepare any discussion of it which you may wish to present. It is issued to the membership in confidence, and with the distinct understanding that it is not to be given to the press or to the public until after it has been presented at the meeting.

The Society as a body is not responsible for the statements of fact or opinion advanced in papers or discussion. (Art. 44 of its Rules.)

As there will be no adequate supply of extra copies, and papers are liable to be read by abstract only, preserve this copy for your use, and

BRING THIS COPY WITH YOU TO THE MEETING.

(Subject to Revision.)

No. 926. *

THE HEAT-ENGINE PROBLEM. †

BY CHARLES E. LUCKE, NEW YORK.

(Non-Member.)

AND PRESENTED BY R. H. FERNALD.

(Associate Member.)

1. A MATHEMATICAL analysis of the different cycles of variation of state through which a mass of gas may pass can give no more than a provisional idea of the value of those cycles for converting the energy of heat into useful power. Such an analysis must presuppose certain ideal conditions that may or may not be possible in practice, and though mathematically we may find that one cycle should convert more of the heat supplied into work than any other, there may be difficulties in the way of practically getting this result. It may happen, for example, that a very complicated large or heavy machine is necessary, or that the

* To be presented at the New York meeting (December, 1901) of the American Society of Mechanical Engineers, and forming part of Volume XXIII. of the *Transactions*.

† For further discussion on the same topic consult the *Transactions* as follows:

No. 843, vol. xxi., p. 396: "An Efficiency Test of a One Hundred and Twenty-five Horse-power Gas Engine." C. H. Robertson.

No. 861, vol. xxi., p. 961: "The Gas-engine Hot Tube as an Ignition-timing Device." Wm. T. Magruder.

No. 875, vol. xxii., p. 152: "Efficiency of a Gas Engine as Modified by Point of Ignition." C. V. Kerr.

No. 879, vol. xxii., p. 312: "A New Principle in Gas-engine Design." C. E. Sargent.

No. 895, vol. xxii., p. 612: "Efficiency Tests of a One Hundred and Twenty-five Horse-power Gas Engine." C. H. Robertson.

required changes of state in the gas cannot be carried out at all, or, perhaps, not fast enough to be useful in a prime mover.

In the general study, then, of the heat-engine problem, we must add to the analytic cyclic discussion a careful consideration of a number of practical questions, the results of which, when allied with the mathematical analysis, will permit of a logical selection of the proper cycle to which we should devote our executive energies; their goal is the production of that prime mover whose source of energy shall be heat, whose medium of transformation of this heat into work a perfect gas, and which shall call for the simplest machine, giving the greatest power in the smallest space with the least metal and under the most favorable circumstances.

2. Every cycle available for transforming heat energy into mechanical energy by the moving of a part against a resistance, must include as one of its phases the heating of the gas in some particular way peculiar to that cycle. This giving of heat energy to the transforming gas presupposes a source of heat which in practice must be a fire. The heat of a fire may be imparted to a mass of gas in three ways:

I. The fire may be placed on one side of a wall through which the heat must pass to the mass of gas on the other side; this may be termed external heating.

II. The fire may be caused to heat a solid mass, which is afterward shut off from the fire and brought into contact with the mass of gas; this is a combination of external and internal heating.

III. The fire may be enclosed and maintained by the mass of gas itself; in this case the gas must be, at least in part, air which will furnish oxygen for this internal combustion.

3. Any system which depends on the heating of the gas by contact with solid matter at a high temperature, must necessarily be slow in operation and involve large masses of gas. For the transfer of heat, the source must be hotter than the receiving mass, and a difference of temperature, for a given rate of transfer sufficiently high to be of practical value, must be greater than the medium of transfer can stand without injury. Consider how hot the walls of a chamber would have to be to heat a mass of gas as rapidly as is done in the gas engine, and the point made above will be clear. Nevertheless, engines with this kind of heating have been built, but, admirable as some of them have

been in conception, they have proved failures as prime movers in competition with others because of the points noted. The engines of Ericsson, Rankine, and the Stirlings are all included in this class, with results that are well known. Ericsson's large engine of 300 horse-power showed a mean effective pressure of about 2 pounds per square inch with a piston area of 600 square feet. The only machine now working with this external heating is the one known as Rider-Ericsson, used in small sizes only for the slow pumping of water.

4. Nothing that this system can do will compare with what may be derived from the use of the internal-combustion method of heating. This internal-combustion heating of a mass of gas will permit of a heating as rapid as we choose, and to any temperature up to a certain maximum. If all the air supplied has its oxygen converted with the fuel to CO_2 , H_2O , etc., there being no excess of either oxygen or fuel, then the mass of gas which, it is true, has changed in chemical composition, but not materially in physical properties, has received the maximum amount of heat obtainable from the combustion of the fuel used. If only a part of the air support combustion and the products be diluted with unused air or by steam, etc., then any desired temperature between the original temperature of the gases and the maximum may be obtained. The problem of heating gases by an internally maintained fire is difficult, compared with the other method of external heating, and this may account for its later application. We might say in brief that externally heating a gas is thermally bad but easily done, internally heating the gas, thermally good but not so easy to do.

5. Heating working gases by internal combustion has been done with coal, oil, and gas. The methods used might be tabulated briefly.

I. With coal :

(a) Air is passed through a coal fire with or without a grate. Cayley, Shaw, and Genty.

(b) A coal fire is moved through an enclosed mass of air. Lord.

II. With liquid fuel not previously vaporized :

(a) The enclosed air acts as a quiet atmosphere supporting the combustion of a jet of oil flame. Diesel.

(b) The air is caused to move past a burner, and in passing

supports combustion, the heated products passing on. Wilcox, Brayton, Nordberg, and Shadall.

(c) Oil is thrown into a hot chamber, there vaporized, and brought into contact with the air, the proportions being so maintained as to make the resulting gaseous mixture explosive. Combustion is of the self-propagated sort. Hornsby, Mietz & Weiss, and Capitaine.

III. With gas or previously vaporized oil :

(a) An enclosed air atmosphere supports a quiet jet of gas flame. Diesel and Gibbs.

(b) Air in motion passes a fixed gas flame as in most atmospheric engines. Wilcox, Weiss, and Otto atmospheric.

(c) Air mixed with gas in explosive proportions is caused to pass a point where the combustion is localized. Brayton, Schmid, Beckfeld, and Reeve.

(d) Air mixed with gas in explosive proportions is enclosed in a chamber, and while at rest burned by self-propagation, after inflammation was provoked by a local ignition. Otto, Priestman, Nash, Westinghouse, and in fact nearly all existing internal-combustion engines.

The above classification leads directly to the broad division of internal-combustion engines into two great classes, the explosive and non-explosive. The term "explosive" we shall apply to all those engines in which a mass of gaseous mixture at rest is ignited at one point, and the whole burned by self-propagation. The other term, "non-explosive," we shall apply to those engines in which the gases are in motion and in that motion pass a point where combustion is localized, and are there heated in the passing. To complete our terminology, we add the expressions "intermittent non-explosive," to those machines in which the combustion is periodically interrupted at the cylinder end as in Diesel's, and "continuous non-explosive" to those in which the combustion is maintained in a chamber, and the hot gases used as required, as in Reeves, Schmid, and Beckfeld.

We have, then, explosive engines ; non-explosive engines with intermittent combustion, or continuous combustion as the different kinds of internal-combustion engines.

6. The explosive engine as developed and perfected, chiefly by Dr. Otto, holds the field to-day, and its very general use has brought out its merits and demerits. It has been, and is to-day, the subject of many researches and experiments, all

tending to perfect it by the discovery of its faults. All this has resulted in its present position, which might be summed up as follows :

It is extremely simple in construction, having comparatively few working parts.

The thermal changes of heating and expansion are all performed in the same place, on a quiet mass of gas, and nothing but the gas is heated.

The best engines—those of rational design—do not differ much in construction and results, and this brings out an important point—that in the handling of a mass of gas to be exploded, we accept a certain inflexibility from which we cannot escape.

7. As a machine, it cannot compare with the steam engine. It is not easy to start, and cannot be worked at widely variable speeds ; its governing is bad, the speed varying at different points of the stroke, but adding up to a fairly constant total number of revolutions per minute ; it has no margin of power and carries an ignition system that once deranged stops the machine ; it is non-reversible ; it has a low mean effective pressure for high-pressure range, hence is heavy ; it can use only one kind of fuel, and that gas, and whether this be produced from oil or coal, it must, nevertheless, be produced outside the natural gas regions. It cannot, and never will be, able to use crude, unrefined oils directly ; it operates under only one fixed cycle.

8. About as much can be said for the explosive type as against it. It has occupied nearly all workers in the internal-combustion field for the past thirty years, and the success attained continues to draw to the problem large numbers of men and an immense amount of capital, and these, working together, must do much for this type in the future. But while this good work goes on, there is no reason why the other types of internal-combustion engines should not receive their share of attention. Some have been built and many proposed ; some were successful and some failures ; but a careful study of what has been done successfully and the cause of failure of the unsuccessful engines would, if no more, show clearly the possibilities of this type. If the difficulties are clearly set forth, the solution will be the easier, and if in the study of the difficulties the solution appears, so much the better.

Of the successful engines of the non-explosive type, there may be mentioned two that easily head the list, the old Bray-

ton and the modern Diesel, and the results obtained from these machines are certainly encouraging. However, before entering into a discussion of the various non-explosive machines, it would be well to make sure of our theoretical grounds.

9. The different cycles of operation that might be performed on a mass of gas are infinite, but there is a limited number which are striking and simple. These are given below.

Let Fig. 1 be a pressure-volume diagram for the Cycle I.

Then we have:

From *B* to *C*. Addition of heat isometrically from atmospheric pressure.

From *C* to *D*. Adiabatic expansion to atmospheric pressure.

From *D* to *B*. Cooling at atmospheric pressure.

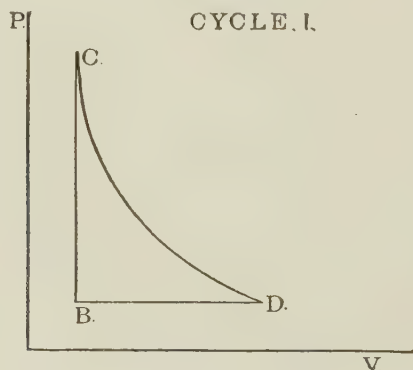


FIG. 1.

In Fig. 2 we have for Cycle I A.:

From *B* to *C*. Addition of heat isometrically from atmospheric pressure.

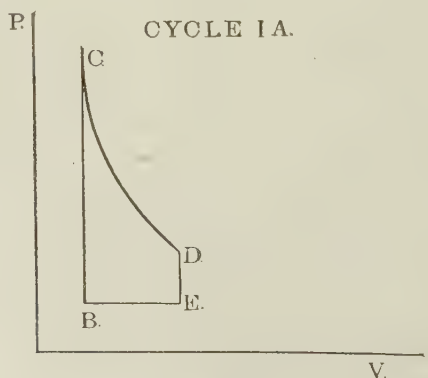


FIG. 2.

From C to D . Adiabatic expansion to a point above atmospheric pressure.

From D to E . Cooling isometrically to atmospheric pressure.

From E to B . Cooling at atmospheric pressure.

In Fig. 3 we have for Cycle IB :

From B to C . Addition of heat isometrically from atmospheric pressure.

From C to D . Adiabatic expansion to below atmospheric pressure.

From D to E . Cooling isothermally to atmospheric pressure.

From E to B . Cooling at atmospheric pressure.

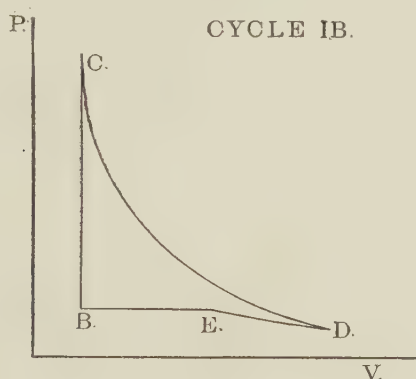


FIG. 3.

In Fig. 4 we have for Cycle IC :

From B to C . Addition of heat isothermally from atmospheric pressure.

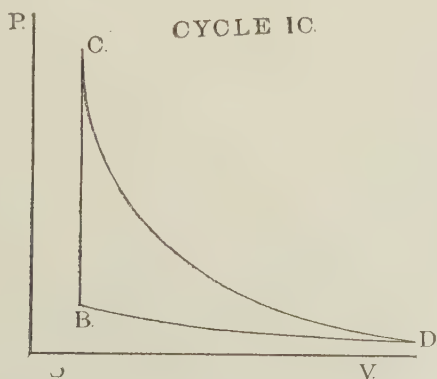


FIG. 4.

From C to D . Adiabatic expansion to a pressure below atmospheric such that we get,

From D to B . Cooling isothermally to the original volume and atmospheric pressure.

In Fig. 5 we have for Cycle II:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isometrically.

From C to D . Adiabatic expansion to atmospheric pressure.

From D to A . Cooling at atmospheric pressure.

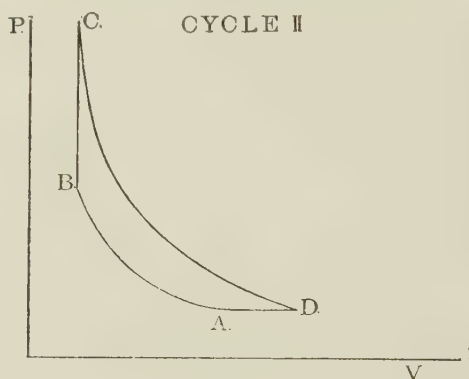


FIG. 5.

In Fig. 6 we have for Cycle IIA.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isometrically.

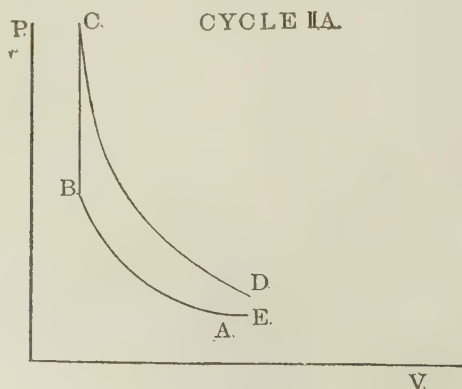


FIG. 6.

From C to D . Adiabatic expansion to a pressure above atmospheric.

From D to E . Cooling isometrically to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

In Fig. 7 we have for Cycle IIB.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isometrically.

From C to D . Adiabatic expansion to pressure below atmospheric.

From D to E . Cooling isothermally to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

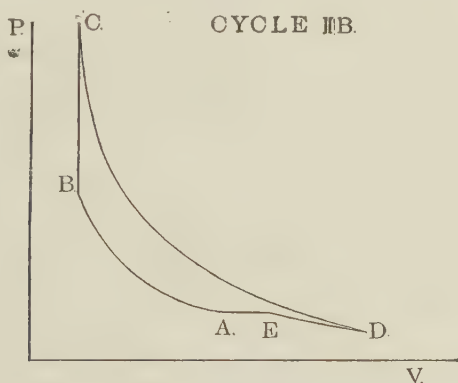


FIG. 7.

In Fig. 8 we have for Cycle IIC.:

From A to B . Adiabatic compression from atmospheric pressure.

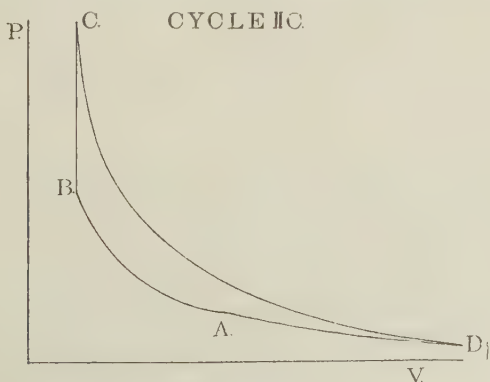


FIG. 8.

From B to C . Addition of heat isometrically.

From C to D . Adiabatic expansion to a pressure below atmospheric such that we get,

From D to A . Cooling isothermally to the original volume and atmospheric pressure.

In Fig. 9 we have for Cycle III:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to atmospheric pressure.

From D to A . Cooling at atmospheric pressure.

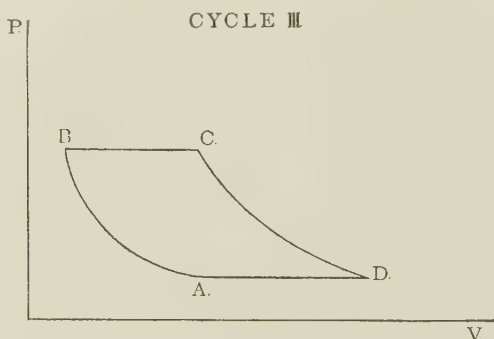


FIG. 9.

In Fig. 10 we have for Cycle IIIA.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to a pressure above atmospheric.

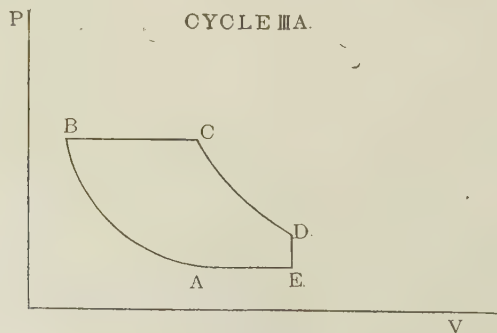


FIG. 10.

From D to E . Cooling isometrically to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

In Fig. 11 we have for Cycle III B.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to a pressure below atmospheric.

From D to E . Cooling isothermally to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

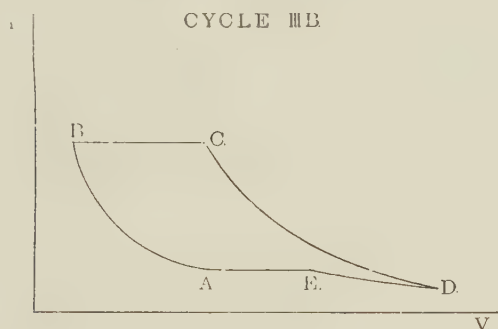


FIG. 11.

In Fig. 12 we have for Cycle III C.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isopiesticly.

From C to D . Adiabatic expansion to a pressure below atmospheric such that we get,

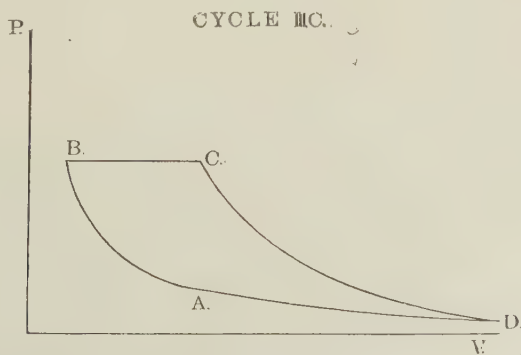


FIG. 12.

From D to A . Cooling isothermally to the original volume and atmospheric pressure.

In Fig. 13 we have for Cycle IV.:

From A to B . Adiabatic compression from atmospheric pressure.

From B to C . Addition of heat isothermally.

From C to D . Adiabatic expansion to atmospheric pressure.

From E to A . Cooling at atmospheric pressure.

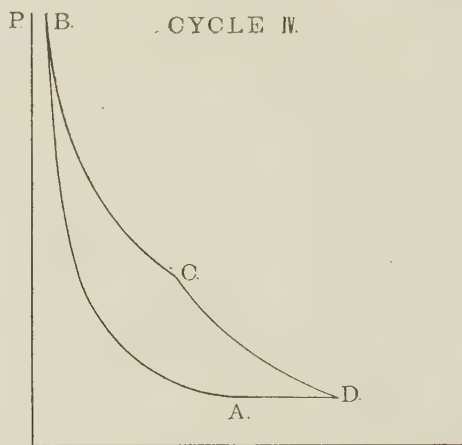


FIG. 13.

In Fig. 14 we have for Cycle IVA.:

From A to B . Adiabatic compression from atmospheric pressure.



FIG. 14.

From *B* to *C*. Addition of heat isothermally.

From *C* to *D*. Adiabatic expansion to a pressure above atmospheric.

From *D* to *E*. Cooling isometrically to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

In Fig. 15 we have for Cycle IVB.:

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isothermally.

From *C* to *D*. Adiabatic expansion to a pressure below atmospheric.

From *D* to *E*. Cooling isothermally to atmospheric pressure.

From *E* to *A*. Cooling at atmospheric pressure.

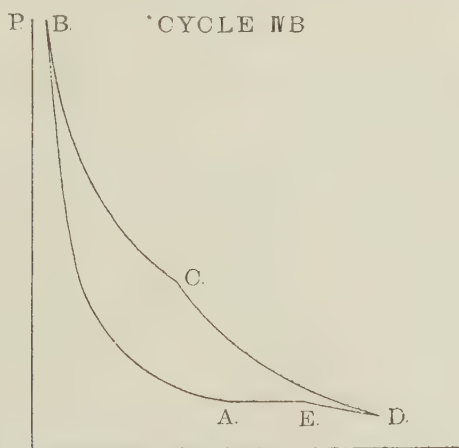


FIG. 15.

In Fig. 16 we have for Cycle IVC.:

From *A* to *B*. Adiabatic compression from atmospheric pressure.

From *B* to *C*. Addition of heat isothermally.

From *C* to *D*. Adiabatic expansion to a pressure below atmospheric such that we get,

From *D* to *A*. Cooling isothermally to the original volume and atmospheric pressure.

Besides these there are various atmospheric cycles—sometimes called vacuum cycles—in which the first step is the heating of the entering charge at atmospheric pressure. Because of their slight importance they are here omitted.

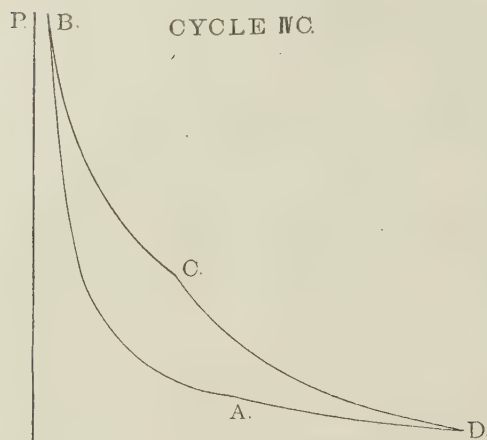


FIG. 16.

10. A very careful mathematical analysis* of all these cycles leads to these conclusions :

(A) Cycle I. and its variations, by reason of its poor showing in efficiency and mean effective pressure as compared with the previous-compression Cycle II., must be set aside.

(B) The atmospheric cycles, by reason of their low mean effective pressure and consequent large volume range, are useless for power purposes as compared with the other cycles.

(C) This leaves as the only cycles worthy of application, II., III., IV., and their variations.

(D) Of the last mentioned, there are three which are peculiar, and these are : Cycle IIA., Otto, heating and cooling the gas at constant volume ; Cycle III., Brayton, heating and cooling the gas at constant pressure, and Cycle IVC., Carnot, heating and cooling the gas at constant temperature.

All these have the same efficiency for the same compression, and should, consequently, with the same heat supplied, do the same work.

The efficiency of each is given by

$$E = 1 - \left(\frac{V_b}{V_a} \right)^{\gamma - 1},$$

where V_a is the volume before compression,

V_b " " after " "

γ " ratio of specific heats, and for air, $\gamma = 1.406$.

* Columbia School of Mines *Quarterly*, Nos. 1, 2, 3, vol. xxii., 1901.

(E) The other cycles, II., IIB. and C.; IIIA. and B.; IV., IVA. and B., can easily be given their proper comparative position by remembering that each is a more or less complete expansion of one of the above three. For example, if in the Otto the expansion were carried to atmospheric pressure, the efficiency would be greater than for the Otto. Similarly with the Carnot, if the expansion were stopped at atmospheric pressure as was first suggested by Diesel, the resulting Cycle IV. would have an efficiency less than the Carnot, and hence less than either the Otto or Brayton cycles.

(F) The other variables entering have the values tabulated for each of the cycles adopted for comparison.

Given the $\left\{ \begin{array}{l} \text{Same mass of gas,} \\ \text{Same heat supplied after,} \\ \text{Same compression.} \end{array} \right.$

There will result for

$\left. \begin{array}{ll} \text{Cycle IIA., Otto} \\ \text{“ III. Brayton} \\ \text{“ IVC. Carnot} \end{array} \right\} \text{Same work done, and hence same efficiency.}$

And, further,

	Lowest.	Intermediate.	Highest.
Maximum temperature.....	Carnot	Brayton	Otto
Pressure range.....	Brayton	Carnot	Otto
Volume range	Otto	Brayton	Carnot
Temperature range	Carnot	Brayton	Otto
Mean effective pressure	Carnot	Brayton	Otto
Pressure range	Brayton	Carnot	Otto
Mean effective pressure	Brayton	Carnot	Otto
Mean effective temperature.....	Carnot	ayton	Otto

The relation of the Diesel to the Otto and Brayton is easily seen, if we remember it as an imperfect Carnot.

11. Some of these variables should be a maximum and some a minimum. For the maximum temperature the Carnot holds first place, but its impracticability yields the place to Brayton. Neither pressure range nor mean effective pressure is wanted by itself, but only the ratio between them, for it is to this ratio that the weight of the engine must be approximately proportional; here Brayton holds the most favorable place.

Volume range should be low, and here first place is held by the Otto. The mean effective temperature should be low, and the Brayton is exceeded only by the Carnot.

The low mean effective pressure of the Carnot, and all other

isothermal combustion cycles, is sufficient warrant for cutting them out of consideration in comparison with the Cycles II., III., and their variations.

We have thus arrived at the conclusion that, theoretically, the last-named cycles only are worthy of further consideration.

Of these the Brayton, III., holds a most favorable position, being surpassed by the Otto only in the position of volume range.

12. In the above, the hypothesis that heat could be added to the gas has been assumed, and no account taken of the means of so doing, but this point needs consideration. If heat be added through walls from a source of known supply, of which we can control and use as much or as little as we please, there will be no alteration in the formulæ or results of the analytical comparison; but the internal-combustion method of heating presents some new questions for solution. First, the air and fuel become carbonic acid, steam, etc., and as to what value of the specific heat should be used, who can say? Second, the chemical change is accompanied by an intrinsic volume change. Third, there may be reasons why the fuel should give out more heat when burned in one way than when burned in another.

13. The only ways of heating by internal combustion that are worth anything for power are the constant volume and constant-pressure methods. On theoretical grounds, we have no reason for saying that, for any particular system of combustion, more heat can be developed one way than the other. The evidence that heat has been added to a mass of gas in an engine is, for the two cases: (*A*) an increase of pressure, and (*B*) an increase of volume. This pressure increase on the one hand and volume increase on the other we can readily observe by indicators, and the results of these observations on a large number of indicator cards show that the increase is not what it should be if all the calorific value of the fuel had developed.

In short, there is in practice abundant evidence of heat suppresion, and whether this be due to radiation, conduction, dissociation or an increase of specific heat, or to an actual non-production of heat is unknown. What we do know and can assert is that the effects on pressure and volume are such as they would be if only a part of the heat supposed to be generated had appeared. The result might be worked up to give a new value to the heating power of the fuel, to be called its *effective* calorific value, or a

new value given to the specific heat, to be called the *effective* specific heat of the process.

14. For constant-volume combustion we have, for H_1 , the British thermal units per pound of mixture,

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = 1 + \frac{H_1}{C_v T_1},$$

where

p_1 = pressure before compression.

T_1 = temperature before combustion.

p_2 = pressure after combustion.

T_2 = temperature after combustion.

C_v = specific heat at constant volume.

This ratio in the general run of gas engines will average about 3.5. In some cases it may reach 4, but I know of no case where it has reached 5. Some values are given below:

Engine.	$\frac{p_2}{p_1}$	Remarks.
Westinghouse	3	On Gas
Otto	4.5	N. Y. Gas
Hornsby Ackroyd	3.5	Kerosene
Nash	4	N. Y. Gas
Clerk	4	Glasgow Gas
Crossley	3	Dowson Gas
Priestman	3.5	Kerosene
Crossley oil	3.5	Kerosene

A general statement, very nearly true, would give these pressure and temperature ratios about 50 per cent. of what the usual values of H_1 and C_v would produce. These figures, while not strictly true for any one case, give a fair average value.

15. The other system of combustion—that at constant pressure—may be observed in the same way. The only indicator card I have of this type of engine was taken from a Brayton oil engine with its smoky fire. The volume ratio, in this case, is quite well given by the relative lengths of the delivery line of the compressor and the admission line of the power cylinder, and is given by

$$\frac{v_2}{v_1} = 3.2.$$

Let us see how this compares with the pressure ratios given. Theoretically,

$$\frac{v_2}{v_1} = \frac{T_2}{T_1} = 1 + \frac{H_1}{C_v T_1},$$

where C_p is the specific heat at constant pressure and the other symbols are as heretofore; combining this with the similar one for the other type we get

$$\frac{H_1}{T_1} = C_v \left(\frac{p_2}{p_1} - 1 \right) = C_p \left(\frac{v_2}{v_1} - 1 \right),$$

or
$$\frac{p_2}{p_1} = 1 + \gamma \left(\frac{v_2}{v_1} - 1 \right). \quad \text{Take } \gamma = 1.4; \text{ and}$$

$$\frac{p_2}{p_1} = 1.4 \frac{v_2}{v_1} - .4.$$

By substitution, when

$$\frac{v_2}{v_1} = 1, \quad \text{we get} \quad \frac{p_2}{p_1} = 1.$$

$$\frac{v_2}{v_1} = 2, \quad \text{“ “} \quad \frac{p_2}{p_1} = 2.4.$$

$$\frac{v_2}{v_1} = 3, \quad \text{“ “} \quad \frac{p_2}{p_1} = 3.8.$$

$$\frac{v_2}{v_1} = 3.2, \quad \text{“ “} \quad \frac{p_2}{p_1} = 4.44.$$

¶ 16. This shows that when a Brayton engine gives a volume ratio in combustion of 3.2, there is evidence of as much heat as would cause a pressure ratio of 4.44 in an explosion engine; hence it would seem that, for the combustion process alone, the Brayton engine, even with its poor fire, was giving evidence of as much heat as the very best explosion engine, and more than can most of them. This point is very striking, and, in order to verify or disprove it, a large mass of data is necessary, which can be collected only after considerable time.

The above point bears strongly on the formulæ of cyclic comparison. The analysis showed that the Otto and Brayton cycles must have the same efficiency for the same heat added; but, if one, by reason of its system of combustion, can take from the fuel more heat than the other, then that one must have the higher efficiency.

17. All non-explosive internal-combustion engines, except the atmospheric types, must provide for three stages: first, the supplying of working gases, which are derived from air and fuel, hence, we need an air and a fuel supply; second, the causing of

the combination of the fuel and air in combustion to raise their temperature, and thereby vary either pressure or volume of the gas, as we desire; third, the utilization of the hot gases thus produced to actuate a mechanism by the action of expanding gas on a moving part.

I. The air and fuel supply may be accomplished in any of the ways known to and accepted by engineers; the results cannot vary much with changes in design of this part, since compressors and pumps are well-known machines.

II. The burning of the fuel in the air supplied offers what is probably the most difficult problem for solution. Its difficulty is attested by the variety of the means proposed and the indifferent success of those that have been tried. When solid fuel was used, as in Cayley's engine, no means, without great complication of parts, were found adequate to cope with the smoke, dust, and distillation of gas from the coal. With liquid fuel, Brayton was troubled with soot, and those burners which have burned clean required a large excess of air. With gas, Brayton also failed, and he was not alone, as no adequate system of burning gas, when enclosed and under pressure, had then been proposed, the trouble being not so much in getting a burner to work under specified conditions, as to get one that would work under wide and sudden variations of feed and pressure.

III. The utilization of the hot gases has been successfully tried in cylinders, and rotary machines have been proposed, including the turbine; though none have appeared on the market, it is inconceivable that there can be any serious trouble to be anticipated in such utilization. The reason of the general failure of the machines proposed is probably the difficulty noted above, for gases at high temperature are used every day in exploding engines with the greatest ease. All of these non-explosive engines may work under any one of several cycles, depending on the relation between the last two processes—the amount of heating compared with the amount of expansion permitted. Here is an important point, for by a simple control of the above relations, by passing air around the fire and varying the cut-off to the power cylinder, we can vary the cycle, hence the work output; thus an engine equipped with means to do this would be able to work at all loads equally well, and be able to pull up to a temporary overload, just as do steam engines. This great elasticity of action is beyond comparison with the

rigidity of the explosive engine. Moreover, the question of available fuel again comes up; anything that will burn may be used, and with it a working elasticity obtained—two desirable results.

Before we examine the details entering into engine construction, let us look at some of the complete machines that have been proposed for carrying out the system, dividing our study according to the fuel used, taking up, first, coal-burning; second, oil-burning; and third, gas-burning engines.

Coal-burning Engines.

18. The first working engine of this class was Cayley's furnace-gas engine. The air was forced into the fire-box, where a coal fire was maintained, and the hot gases used in a cylinder. This engine worked on what has been called the Brayton cycle. Rankine says of it: "The cylinder, piston, and valves were found to be so rapidly destroyed by the intense heat and dust from the fuel that no attempt was made to bring it into use."

In the United States, Philander Shaw proposed the engine of Figs. 17 and 18 in 1861. Air from a pump cylinder passes more or less through a coal fire, becoming heated in its conversion, and finally with increased volume working in a larger power cylinder. The fire-box is provided with a grate, *c*, and is lined with brick *J* (Fig. 17), fuel is fed through *H* (Fig. 18), and is moved by a piston head *g*. The furnace has openings, *a*¹, below the grate, and others, *a*², above the fuel. Two single-acting cylinders are connected at 180 degrees to one shaft. Each cylinder is in two parts: the upper, *A*, is finished to a fit with piston *D*; the lower part, *B*, is left rough, as the lower part, *F*, of the piston does not touch it. The top of the piston has a trunk, *C*, which acts as an air-pump cylinder with the main cylinder casing. The motion of the air is indicated by the arrows, and the proportion that enters the fire-box above or below the fuel is controlled by a valve *r* (Fig. 17). The heated air and gaseous products of combustion pass into the cylinder through valve *p*, and the exhaust passes out through a heater for incoming compressed air. A little flange, *s*, is placed to catch dust and a groove, *v*, for oil.

19. All parts where radiation is likely to occur are jacketed by the incoming air. The working part of the piston fits loosely, and at a point just above the highest position of its bottom is an

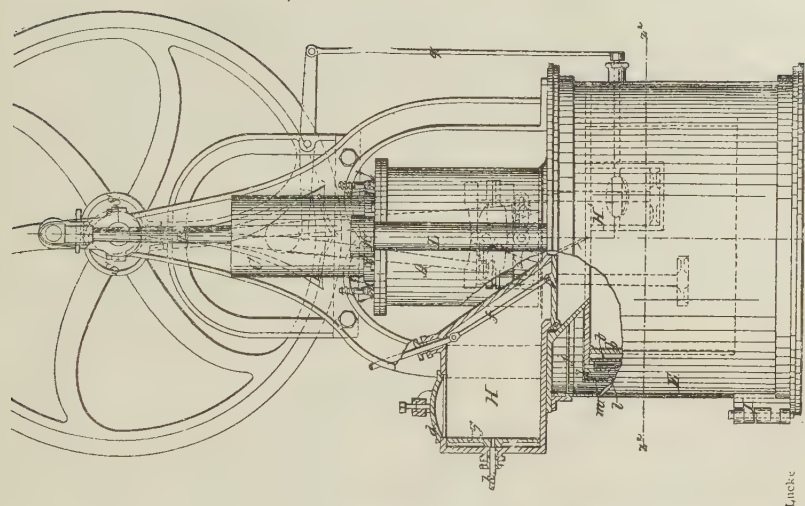


FIG. 18

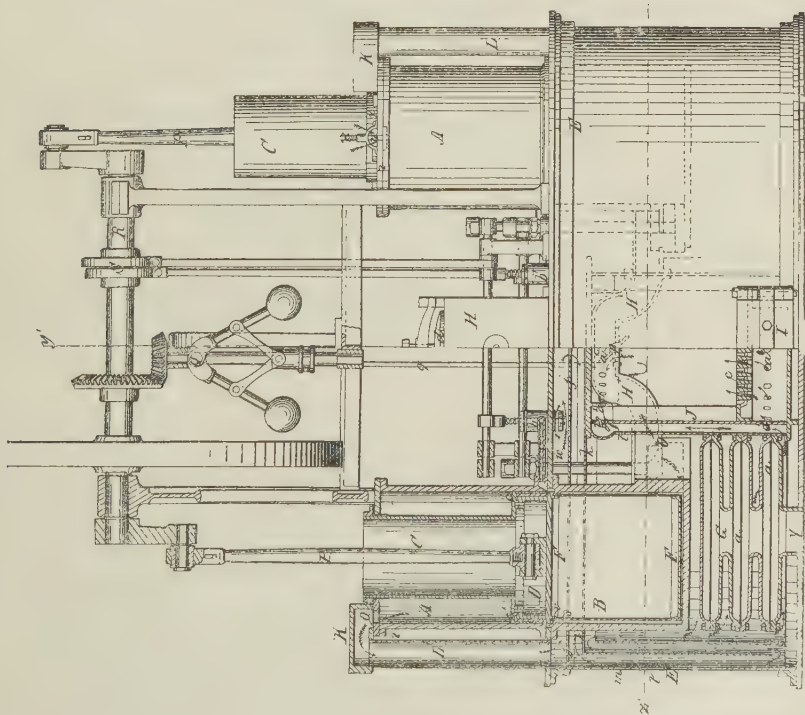


FIG. 17.

annular space kept filled with cool air to prevent overheating of the working faces. Governing may be made two-fold : first, the amount of heat added to the air can be regulated by sending more or less through the fire ; and, second, the power may be directly controlled by the main cylinder cut-off. The air receives heat really from one source, though in two places—the one source being the fire, and the two places the exhaust-warmed pre-heater, or regenerator, and the fire-box. A hand pump is proposed to raise the internal pressure for starting.

20. The engine proposed by Henry Messer in 1863 is shown in Figs. 19–21. The air pump is double-acting, and the power

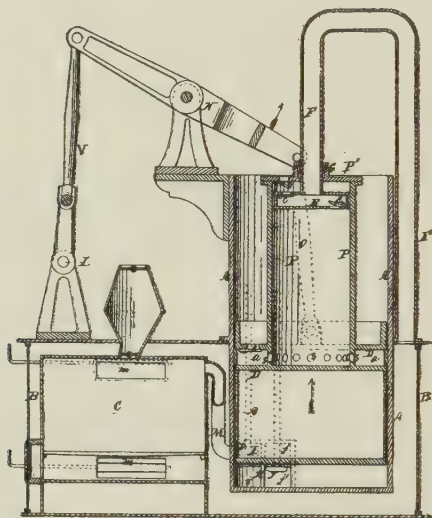


FIG. 19.

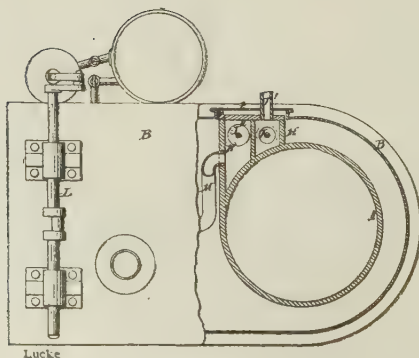


FIG. 20.

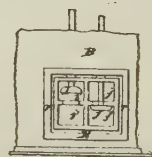


FIG. 21.

cylinder single-acting, hence there are two stages : 1. On the down-stroke air is compressed and heated at decreasing volume ; 2. On the up stroke the air, previously compressed and heated, is sent through the fire and thence to the working cylinder. When the up-stroke begins, the high-pressure gases in the reservoir begin to expand at the same time that the air in the pump begins to increase in pressure, and, finally, when the increasing pressure in the pump equals the decreasing pressure in the receiver the second cylinderful of air becomes available. In its passage the air may take up enough heat to maintain p constant, T constant, or neither ; it is impossible to say, hence

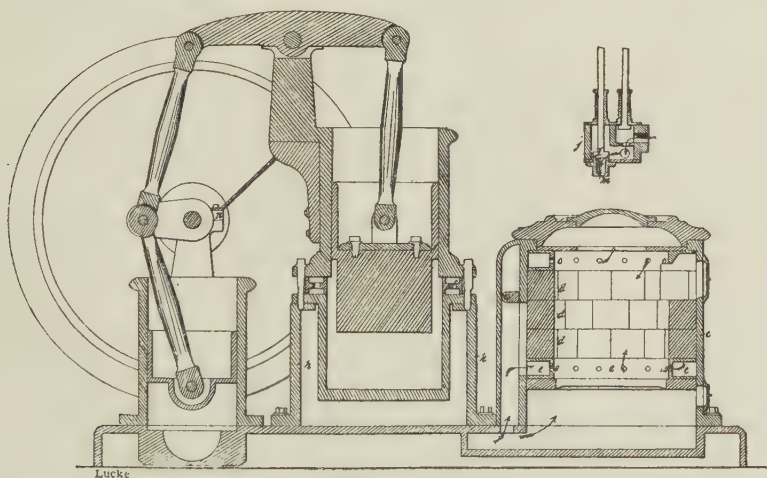


FIG. 22.

the cycle is indeterminate. Later Messer suggests some changes in valve construction to prevent overheating, also in governing by throttling between the fire and air receiver.

21. Cyrus W. Baldwin proposed the engine of Fig. 22 in 1865. The engine cylinder has between its upper cool part and lower hot part an *H*-shaped water passage *f* in accordance with his belief that more trouble with overheated working faces will be caused by heat conducted through the metal cylinder walls than by contact with the hot gases. He provides for distilling gases from the coal fire by supplying an auxiliary fire beyond the main furnace. He says : "Part of the fuel in the large furnace is changed by the heat therein to volatile gases, which do not burn when generated, but will burn if, while they are hot, they are

brought into flame. To supply such a flame through which all the products of combustion from the large furnace must pass, a small furnace is supplied, and the results which follow its application are found in practice to be highly beneficial."

22. In the engine of L. A. L. Söderström, 1869, a radical change of arrangement is proposed, as shown in Fig. 23. The

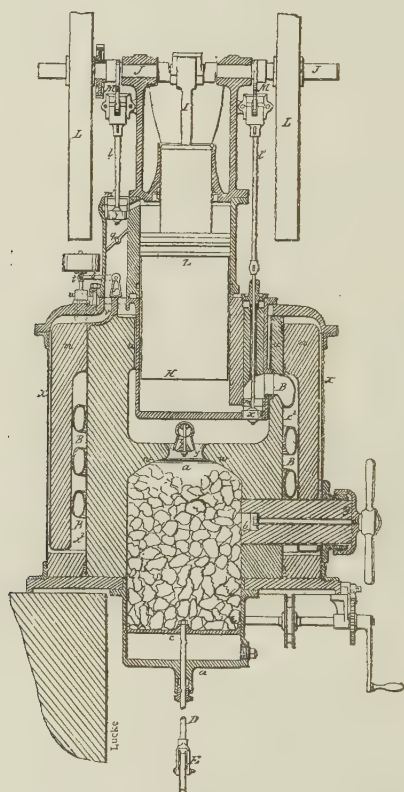


FIG. 23.

upper part of the cylinder acts as the compressor and sends air down and around the casing, the air passing the exhaust reheating coil *B*. After this reheating by contact with the exhaust coil the air is sent out through the opening *yx* on top of the fire at *a*, the top combustion helping to prevent coking and distillation, but adding trouble from dust and ash. Governing is effected by a split current and variable cut-off.

23. Thomas M. Fell in 1880 proposed a very interesting though complicated machine (Fig. 24). An air compressor, C^1 , sends air through the pipe M^1 to the fire at A . Exhaust gases from the power cylinder A^1 are thrown first around the tubes D^1 , and then around the water-cooled tubes E^1 , and are returned by the pump B^1 , through the tubes D^1 , and back to the hot chamber T through the valve H^1 . Accumulation of gases in the closed system is prevented by the blow-off G^1 , arranged to maintain a constant pressure on the high-pressure side of the system. An attempt is made to keep a two-part system, one

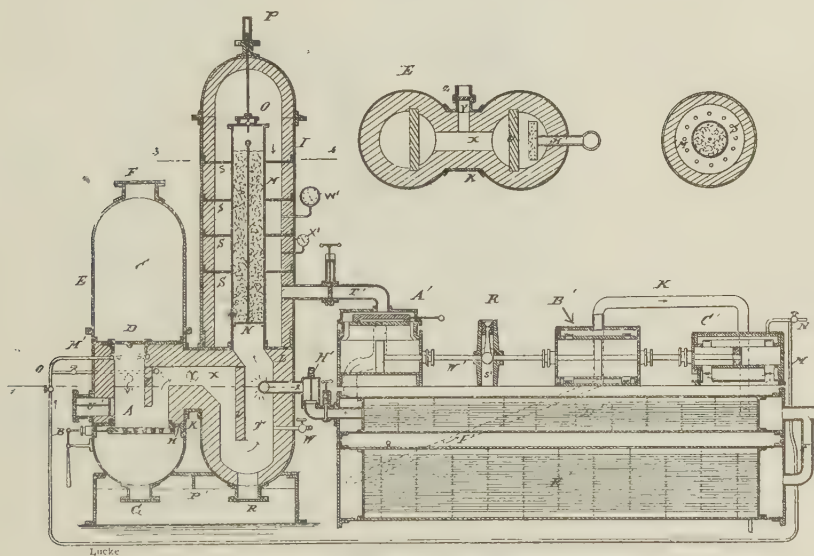


FIG. 24.

high-pressure heating, and the other low-pressure cooling. This system would probably give the results of Cycle III C., or one somewhat similar.

24. Hiram S. Maxim in 1884 proposed a machine as shown in Fig. 25. Air is drawn by the right-hand side of the piston from the space A^1 , which in turn gets its supply from the atmosphere by a throttling slide T . This space A^1 has a diaphragm R so arranged that a partial vacuum will cause the slide valve N to by-pass air around the fire. In its normal operation, or when running slowly, most of the air passes through the fire and is heated. At the time the air is passing into the receiving chamber no air is entering the working side of the cylinder. Hence

the heating of the air during this stage must take place at decreasing volume and increasing pressure. When this is finished, the valve *V* opens and gases enter on the impelling side of the piston. The tendency now is to decrease pressure, but air in the space *G'* will pass the fire, tending to uphold pressure; hence there is a second heating at uncertain pressure, giving an indeterminate cycle.

25. Lucien Genty in 1893 proposed the engine of Figs. 26-28. This engine has the usual single-acting cylinder with elongated

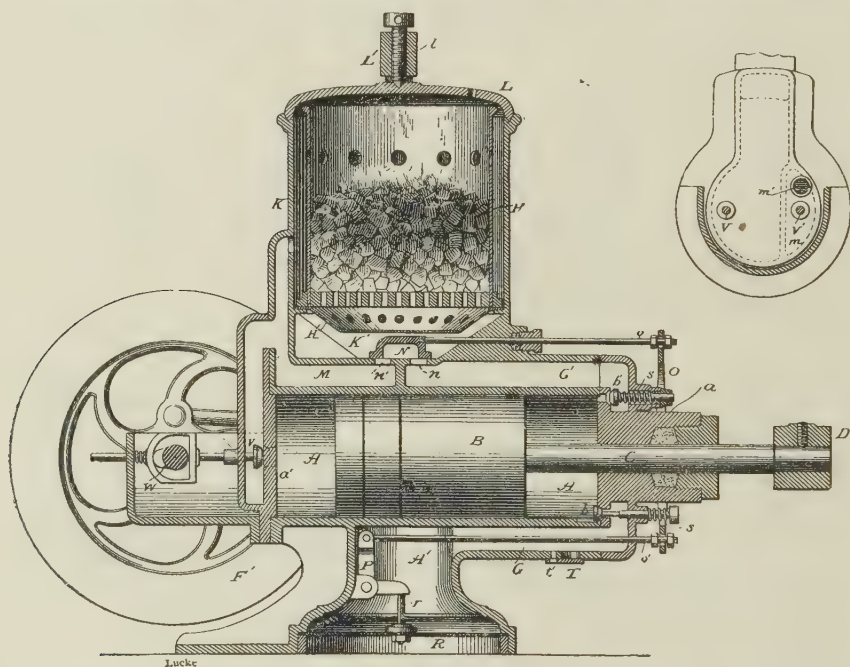


FIG. 25.

piston. Air is drawn from a chamber under the floor to obviate the noise of suction. The air thus received by the water-jacketed compressor 10 is sent through the valves 17 into the hollow bed 4, and thence past the ribbed-coil preheater, here to be warmed by the exhaust. The preheater is arranged to take up expansion, and exhaust gases are prevented from touching the metal by brick lining. The partly heated air enters admission valve 36 by pipe 35. Valve 36 is double balanced, and held to its seat by a helical spring, and is actuated by a cam on the horizontal shaft.

26. The valve gear is arranged to govern by varying the opening of valve 36. A weighted piston controller regulates the proportion of air admitted to the two passages 41 and 42 after passing the admission valve 36. Thus the method of governing is two-fold ; 1, by a variable cut-off to the power cylinder and combustion chamber, and 2, by means of keeping the pressure of the air admitted initially constant. Passage 42 leads above the fire and 41 below.

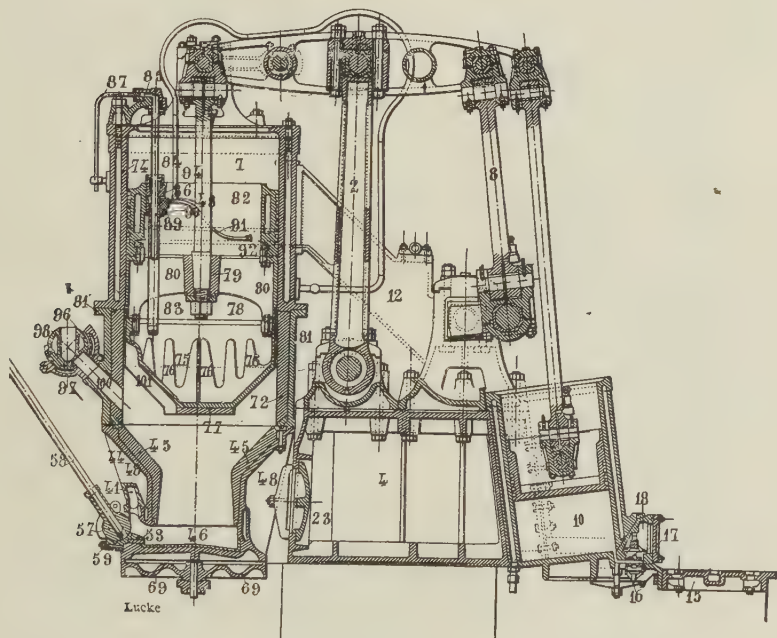


FIG. 26.

The combustion chamber is a cast-iron funnel lined with brick, enlarged at the bottom ; the coal is thus burned without a grate. The lower and hottest part has a water-jacket, 49. Means for charging and cleaning are provided, and a ball-and-socket swivel, 57, carries a stirrer, 58, which is removable. After passing the admission valve and reaching the fire, the air is first heated and afterwards expanded in the presence of the fire under the piston. The motor cylinder is of two parts ; the lower, 72, is lined with fire clay, the upper, 74, is bored true and water-jacketed. The lower part of the piston has air-cooling ribs, 76, and the rod is rigidly connected to the yoke 78, leaving no lubricated joint within the piston. The piston fits loosely in parts

75 and 78, and tight in part 74. Besides having the water-jacket on this part it may be further protected from heat and dust by a groove, feeding air down to the combustion chamber; to do this air is provided by a small pump on the piston at 89. A flexible pipe provides water to cool the piston interior.

27. After being heated and expanded the gases are discharged through the self-cleaning valve, 110. This valve is hollow and water-cooled, as is also its casing, 116. Compressed air seats the valve, and mechanism raises it. Injury from sparks, etc., is

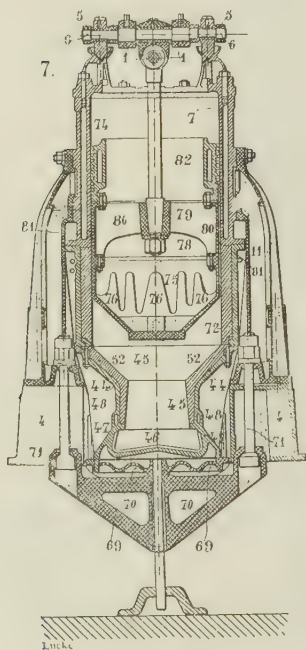


FIG. 27.

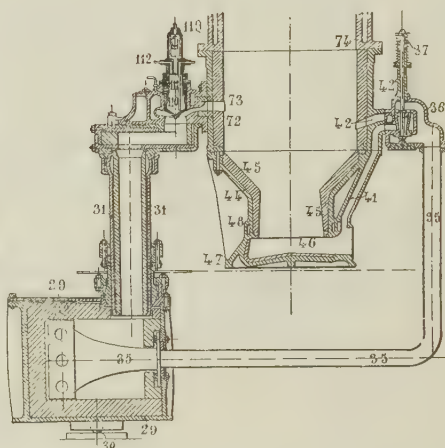


FIG. 28.

prevented by drawing the valve up in its sleeve casing. To protect the seat it is made narrow and cleaned by escaping air. Exhaust gases are discharged through the regenerator, or pre-heater. This machine, though very complicated, presents many suggestions that may be of value.

It will be observed that the combustion of coal calls for a great complexity of parts and functions, and this must be so. We have, as one of the greatest troubles, the impossibility of regulating the heating power with time of a coal fire, and there is the inevitable dust and ash. This makes the use of oil and

gas with the necessarily simplified apparatus almost mandatory in internal-combustion engines, especially those of moderate size. While here we should have none of the coal troubles, we will meet others incidental to the feeding of fuel as it is required, and only for the instant that it is required.

Oil-burning Engines.

28. In 1865 Stephen Wilcox, Jr., proposed an oil burning engine, shown in Fig. 29. He said, "The extraordinary devel-

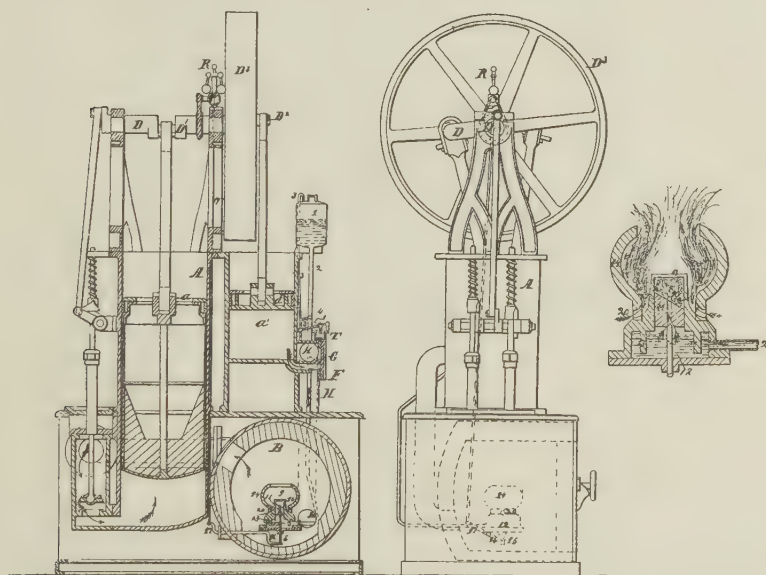


FIG. 29.

poment of what is known as petroleum oil and the several products obtained therefrom, makes it practicable to produce and work very small engines on this plan."

A, the working cylinder, and *a'*, the pump, differ in no part from what we have seen in the coal-burning engines. The means for feeding and burning the oil are, however, new and of especial interest. Fuel is fed from an elevated tank 1 through pipe 2; the interior pressure of the furnace is balanced on the surface of the oil by pipe 3. A stop-cock, 4, is provided, which is arranged to shut off oil by a piston and links, when the furnace pressure exceeds what is desired; this serves to govern

the machine. Oil flows into a vaporizing reservoir, 12 (Fig. 33), having lugs, 13, to conduct heat from above. The upper part of this reservoir is cylindrical and provided with holes, 10, and fitted with a loose part, 11, with inclined top and a hole in the centre matching the movable pin, 14, actuated by the governor. This pin and hole act to close the vapor outlet. The special construction of the burner is intended to give a constant velocity of efflux to the vapor.

This combustion may be classed as the burning of a jet of vapor in an atmosphere of air, the air about the flame being kept fresh only by convection.

Governing is effected by a double means; the fly-balls act to shut off vapor and increase of pressure cuts off the fuel.

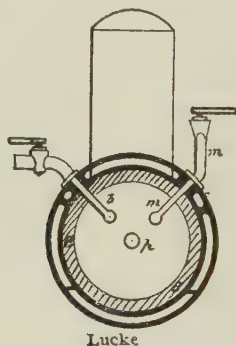


FIG. 30.

29. A. H. De Villeneuve in 1872 proposed an engine in which he provided a combustion chamber, having a platinum rose, *p* (Fig. 30), on which impinge jets of oil vapor from *b* and air from *m* provided by pumps. He thus expects to obtain a combustion to heat the mixture.

George B. Brayton in 1872 proposed and built an engine that was very complete and fairly successful. Fig. 31 is a general view and Fig. 32 his oil burner.

Air is compressed in the single-acting pump, which has a volume one-half that of the power cylinder. The compressed air passes from the constant-pressure receiver through pipe *D* and over the absorbent material *e*, through which the fuel is fed by a pump. Here it takes up vapor and the mixture passes the wire-gauge grating and into the cylinder, where it burns.

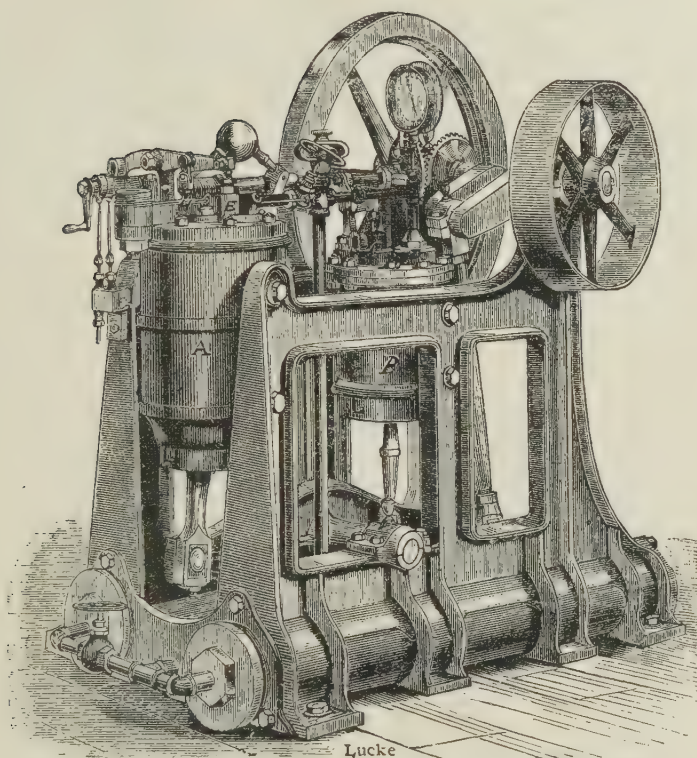


FIG. 31.

Means are provided to prevent entirely shutting off the air from the power cylinder, and thus there is kept constantly burning a small flame which increases for the power stroke. Governing is effected by a variable cut-off to the power cylinder.

The power cylinder is water-jacketed, and no trouble is experienced through overheating. A safety valve is placed on the reservoir.

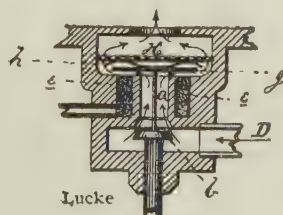


FIG. 32.

30. Joseph Hirsch, in 1874, proposed the engine shown in Fig. 33. An air pump, *L*, supplies air to a cylinder, *H*, to take the place of converted air which is periodically expelled; *J* is a regenerator connecting the two cylinders, *A* and *H*; *N* is a water chamber for cooling gases. Fuel is injected at *b*². When the piston *C* moves down, air from *G* is sent over through the regenerator, partly heated here and then further heated by fuel at *B*². On the up-stroke the products of combustion pass over the regenerator to *H*, being thus doubly cooled, first by the regenerator and second by the injected water. Part of the products of combustion escape at *k* and are replaced by fresh air. No means for ensuring the combustion of fuel in the atmosphere of air and products of combustion are provided.

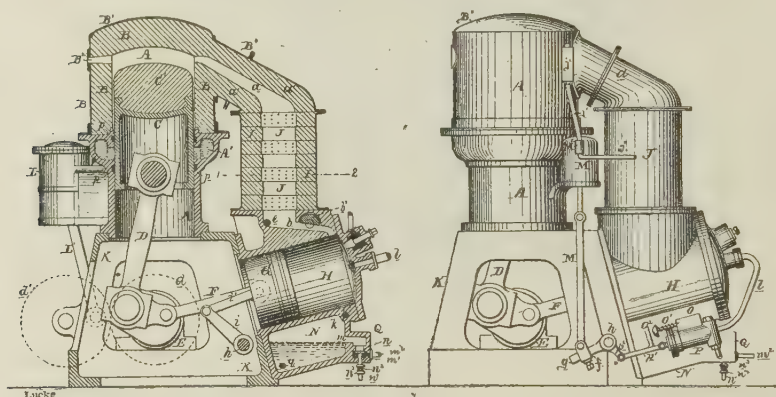


FIG. 33.

31. Stephen Wilcox, in 1885, proposed the engine of Figs. 34 and 35. The power cylinder, *A*, is double-acting and tandem-connected to an air compressor, *B*. Plates of non-conducting material are applied to piston and cylinder heads to keep the gases admitted as hot as possible. Cylinder walls, where lubrication is necessary, are water-jacketed, and water-cooled rocking valves of the Corliss type are provided; air is sent to the exhaust preheater *C* from the pump before passing to the power cylinder, and is, therefore, first warmed and later heated by combustion, in its passage over the gauze grating, *S*, where it meets the liquid fuel. The valve mechanism permits varying the cut-off and reversing.

A reciprocating tube carrying a lamp works in each port *r* and serves as an igniter in connection with an exterior relighter.

The receiver is first charged with air by a hand-pump, and a little fuel sent to the burner by hand. Valve 24 in the escape pipe 22 is closed and a torch applied to the burner through *w*. The engine may now be started by opening the main stop-valve, 28, and automatic action begins.

32. Rudolph Diesel, in 1892, proposed the oil-burning motor shown in Fig. 36. A single-acting cylinder *C* carries the plunger

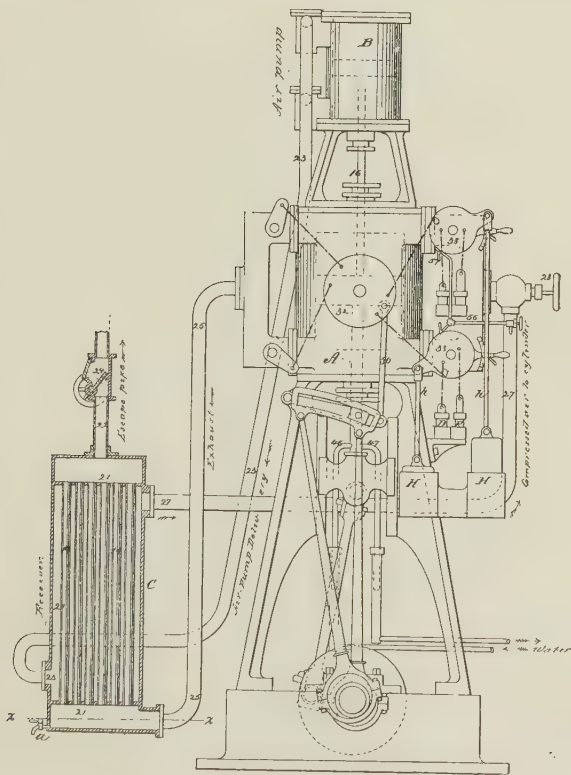


FIG. 34. |

P, air-valve *V*, and fuel-valve *D*. High compression of the air is followed by fuel injection and later by expansion. The temperature developed by the compression must be sufficient to ignite any fuel thrown into the air. Later engines vary somewhat in detail, but the principle of operation is the same. Gaseous and powdered solid fuel can also be used.

It is obvious that the quantity of fuel injected per stroke will determine the cycle. If only sufficient is admitted to keep *T*

constant, we have Cycle IV., or some of its variations; if enough heat results to keep p constant during combustion, we have one of the Cycles III.

33. B. V. Nordberg and C. E. Shadall proposed in 1895, not a complete engine, but a system of operating engines by internal combustion; the apparatus is shown in Fig. 37. The products of combustion are to be used in any way deemed advisable.

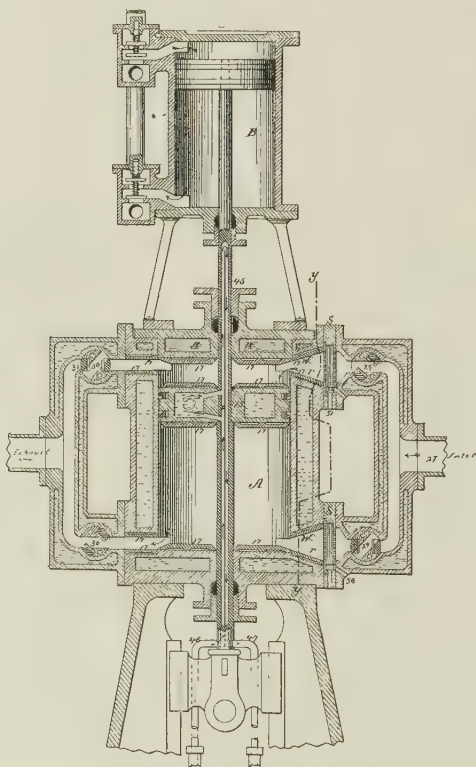


FIG. 35.

A source of air supply and means for using the products are assumed. Oil is fed from a tank B by water displacement, the same water-jacketing the combustion chamber C . This combustion-chamber jacket may add steam to the products at c_1 . The air current from the compressor acts (1) on the surface of the water A ; (2) at the oil atomizer E ; (3) at the lamp I ; and (4) with reduced pressure at the opening c in the burner. The

atomizer is fed with oil through the pipe *b*, and the float b_{13}^{33} prevents, by proper specific gravity, an overflow of water. The oil passes up to the nozzle, where it meets an air current and is there sprayed into the chamber *C*¹; the spray meeting a flame jet from the lamp at *i* in an atmosphere of air provided through *c*, is enabled to burn. Increased pressure cuts off the oil supply

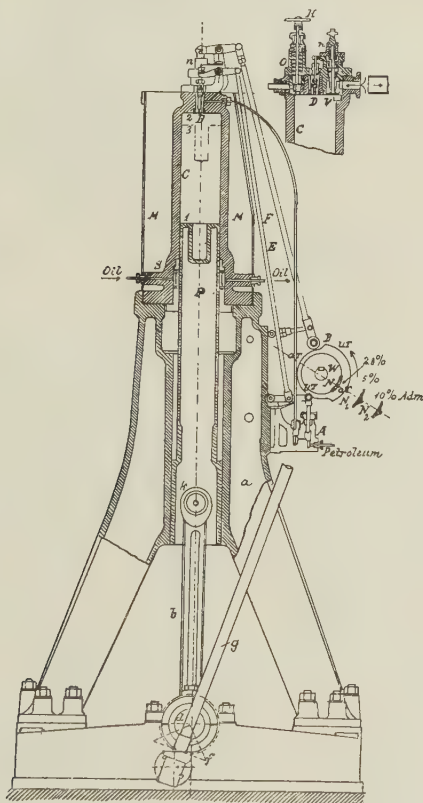


FIG. 36.

Gas-burning Engines.

34. The operation of engines of the class we are considering by a gas combustion offers what would seem to be the simplest solution, but in reality the difficulty is much greater than might be supposed. Of course, no trouble will be experienced with dust, vaporizing of oil, or soot from imperfect oil combustion, but there appears the difficulty of finding a burner which will

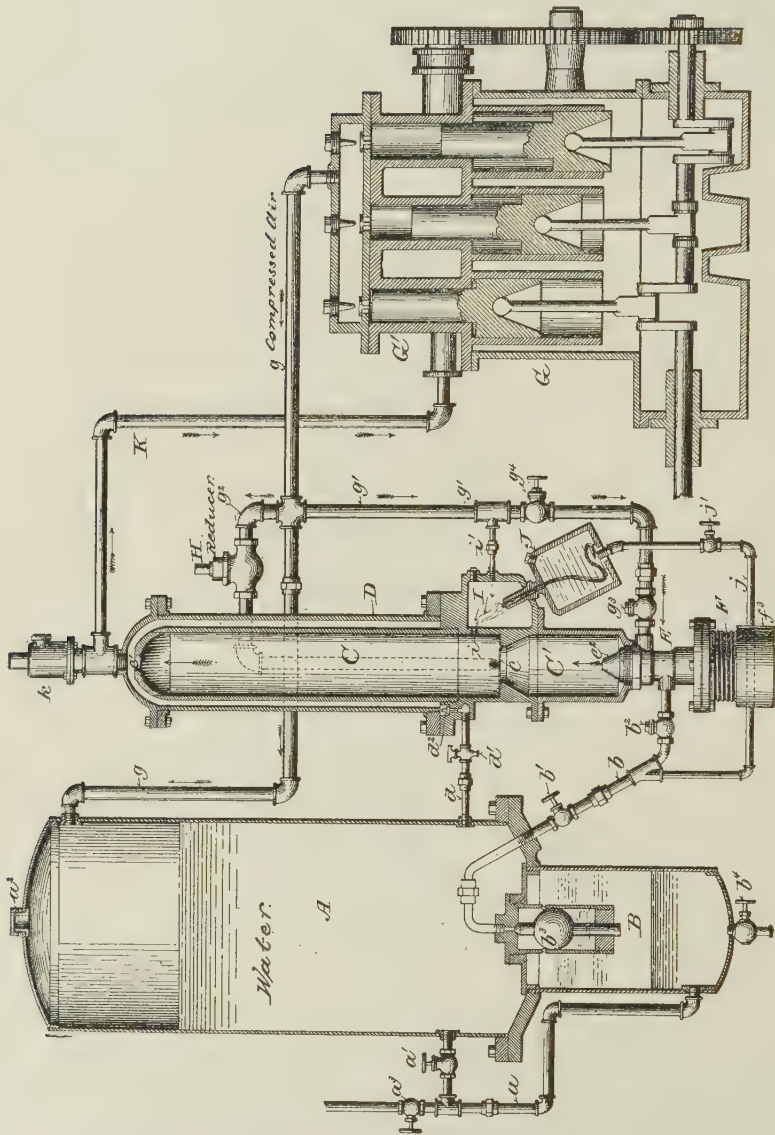


FIG. 37.

completely consume the gas without excess of oxygen under the widest possible range of conditions. The engines proposed differ chiefly in the means proposed for accomplishing this gas combustion.

Stephen Wilcox, in 1865, proposed the machine of Fig. 38. Air and gas are supplied through pipes *K* and *J* from feed-pumps *G* and *H* to burner, and the mixture is ignited at *R*. When the gas is to be supplied by vaporizing oil, the exhaust-heated vaporizer *N* is introduced. *A* is the power cylinder, *B* a changing cylinder, and *F* a regenerator. When *b* descends, the valve *M* allows cold air to fill the top of *B*; this air, on the up-

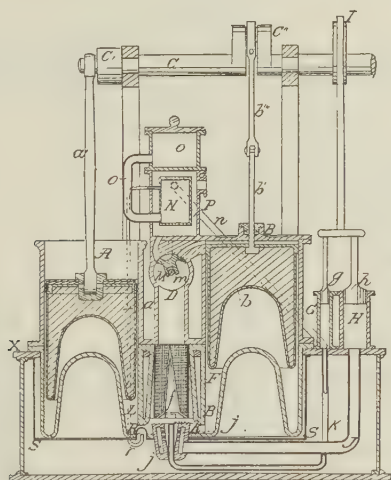


FIG. 38.

stroke, moves through the regenerator and burner, which at this time delivers its mixture; this double heating raises the pressure in the system, and the piston rises. Then both descend and exhaust some gases into *P*. The engine is thus operated not solely through the heating of products of combustion, but by the heating of a mixture of these gases with pure air.

35. Albert Schmid and J. C. Beckfeld proposed, in 1889, the system of obtaining hot gases through internal combustion of air and gas by the apparatus of Fig. 39. Gas and air are supplied by pumps to tanks *A* and *G*; they are mingled in an injector chamber *S*²; thence passing through the perforations *t*, they are burned at *S*. To maintain a difference of pressure in

the air and gas mixture is admitted through *C* and ignited, the combustion operating on the gauze until the mixture is in turn shut off, when adiabatic expansion begins.

37. Sydney A. Reeve, in 1897, proposed the apparatus of Fig.

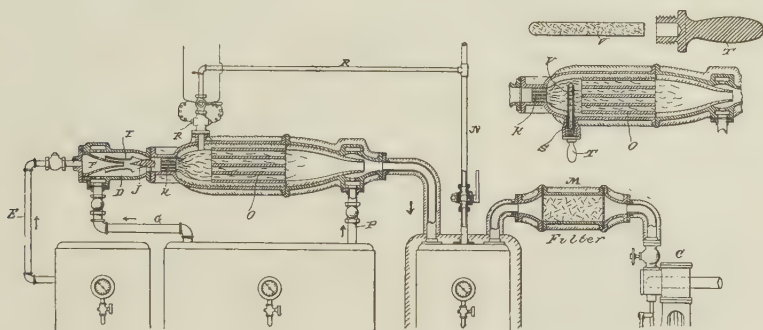


FIG. 40.

44 to obtain by internal combustion working gases to be used in an engine. He lays stress on two points: one, relating to the combustion, that the proportions of fuel to air shall not materially vary; and the other, the reduction of the

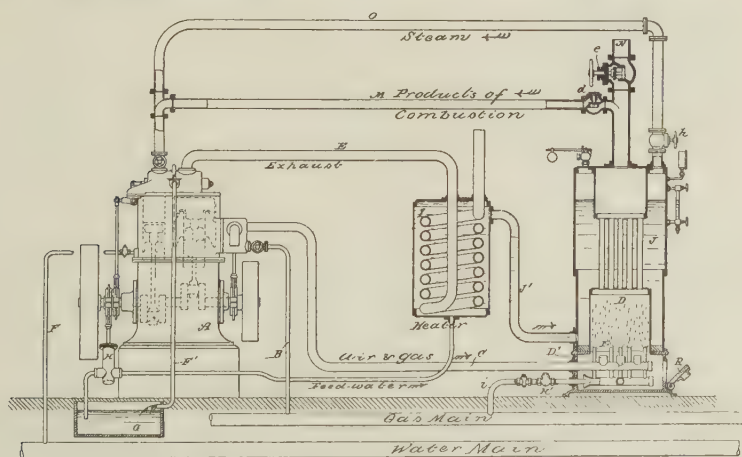


FIG. 41.

temperature before sending the products of combustion to the engine.

Both air and fuel are to be supplied by separate pumps, and the proportions regulated by maintaining the pressures in

the two receivers, C and C^1 , equal by water seal and float valve, and by passing these gases of equal pressure through a double-ported valve D of proper areas. The pressure in C is controlled by that in C^1 , and that in C kept above that in the combustion chamber by the loaded check G . This also permits mixing fresh air with the products of combustion if more than is wanted for combustion is available.

The products of combustion pass through water so supplied as to keep a given quantity always on hand and at the boiling point

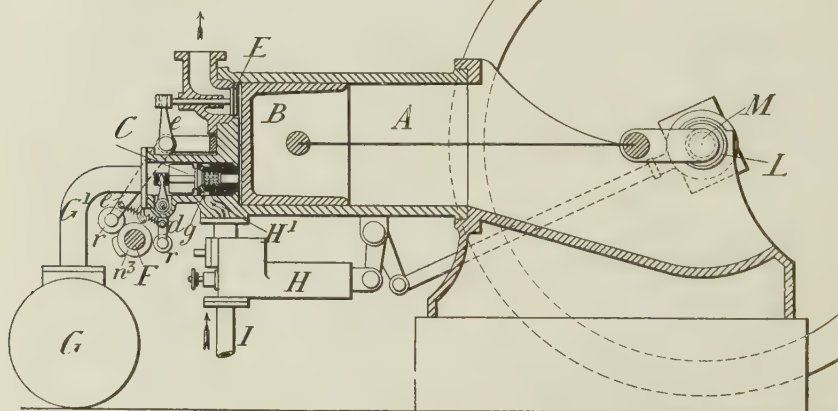
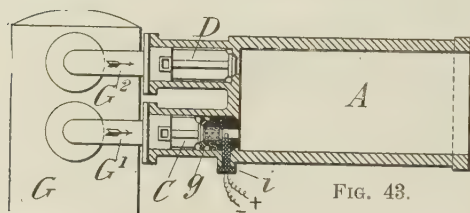


FIG. 42.

corresponding to the pressure, so that the hot gases will, in parting with their heat, evaporate water and be themselves cooled to the temperature of saturated steam at their pressure. Of course, any feed-water supplied must be heated before evaporation, but this only has the effect of decreasing the rate of evaporation without stopping it.

Another device proposed for equalizing pressures in the air and gas receivers is to let fuel pass through a flexible-walled bag suspended in the air tank.

38. The cooler and burner might also be arranged as shown

in Fig. 45. The regulator valve has a spring-balanced diaphragm, 4, which actuates a similar plunger, 6, by rod 5, the plunger moving in a perforated sleeve. Gas enters through tube 12 in the centre and air through 10, the mixture passing an igniter at *b*. A liquid seal is provided here for maintaining a decrease of pressure between supply and discharge valves.

39. Lucius T. Gibbs, in 1897, proposed a system (Fig. 46), in which the motive power shall be air admitted to the power cylinder from a source under pressure, and when the pressure after cut-off has become so far reduced as to reach that of a

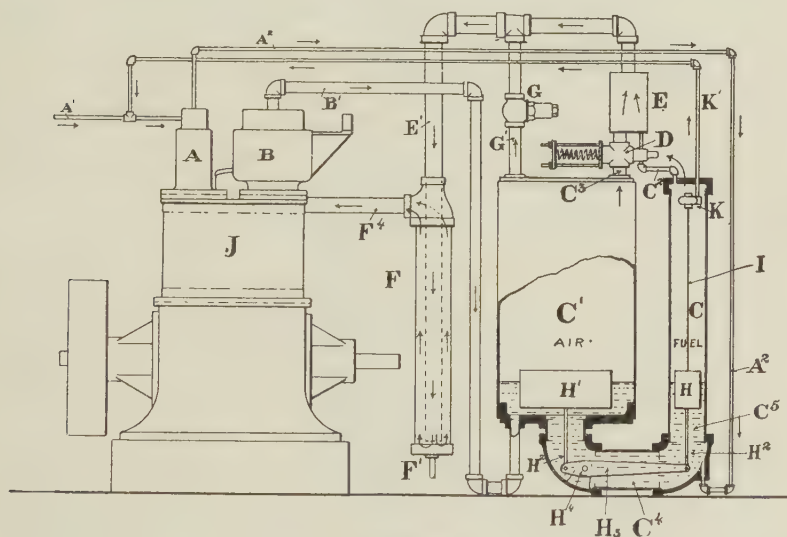


FIG. 44.

stored mass of gas maintained lower than the air supply, the gas will enter and be ignited, thus tending to keep up the pressure during the expansion. Thus the adiabatic will be raised to perhaps an isothermal or higher.

40. If we would trace any line of progress through these machines, we could not make our division according to the fuel used, as at times engines burning all kinds of fuels have been suggested simultaneously. To be sure, before the possibilities of petroleum were known, the principal fuel was coal, and, naturally, in early engines coal fires predominate; though it would seem that they would gladly be dropped for oil and gas, yet they were not, and continue to appear from time to time

and means for transferring them from a hot part to a cold part, and so varying their internal pressure, but the hot part is here provided for not by a hot plate, but by a short flame injection, or passage over a fire. The operation was to depend chiefly on the alternation of hot and cold in the same mass.

41. Later this system was developed by injecting more and more fuel, or by causing the mass to pass entirely through the fire, necessitating more fresh air for the next time and calling for an exhaust; finally we have a regular admission and exhaust, the gases passing continuously in the same direction, and no alternation of heating and cooling being attempted in the system. Any cooling that is to take place must occur outside

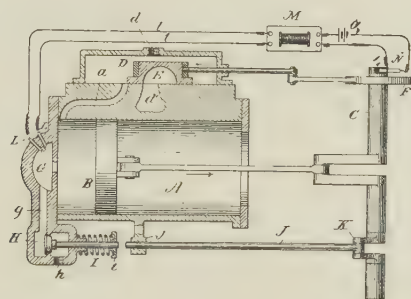


FIG. 46.

the machine in the atmosphere, and the resulting contraction of the gases forms no part of the working cycle.

42. The engines presented represent by no means all of those suggested, but are selected from a very large number to show the principal ideas advanced in the past. By studying them we can reach an understanding of what points may be accepted as solved, and what are still open for discussion and research.

It will be observed that these non-explosive internal-combustion engines may be divided into (I.) intermittent internal combustion, as in Genty's, Brayton's, Wilcox's, Diesel's, etc., in which the air and fuel pass the power-cylinder valves before combustion; and (II.) continuous internal combustion, as suggested by Schmid and Beckfeld, Reeve, and Nordberg and Shadall, in which the hot gases are continuously produced, and afterward utilized by passing through valves to the power cylinder, or by expanding through a turbine without valves.

43. Different ways of doing the same thing by varying details have been advanced, and it may be well to bring these together in some cases for comparison.

Cylinders :

Both single and double acting, jacketed and unjacketed are found. Air and power cylinders may be independent, or the two operations of compression and expansion performed in opposite ends of the same cylinder. When this is done, the cylinder may be of the same diameter throughout, or reduced at one end. Independent cylinders have been connected by beam, separate cranks on the same shaft, and in tandem. In most of the intermittent-combustion type the piston fits a part of the cylinder only, and is loose in the hot end ; in this case, heat is prevented from reaching the working faces by prolonging the piston, and by blowing cold air or steam down around the loose part toward the hot end.

44. Igniters :

With engines using oil or gas, where combustion may be entirely interrupted for a time, some form of relighter must be provided. This may be

- I. A plate heated by an external jet from outside. Nordberg.
- II. A platinum rose, with mixture impinging. Villeneuve.
- III. High temperature of compression. Diesel.
- IV. A plate or wire electrically heated. Wilcox.
- V. Introduction of incandescent solid. Schmid and Beckfeld.
- VI. Electrodes with spark gap. Babcock.
- VII. Introduction of auxiliary flame continuously. Nordberg and Shadall.
- VIII. Introduction of auxiliary flame intermittently. Wilcox.
- IX. Constant-burning flame in cylinder. Brayton.

45. Use of water :

- I. In compression to cool air.
- II. To be evaporated by products of combustion by contact and adding steam produced.
- III. To cool expanded gases and produce partial vacuum.
- IV. To cool hot parts and be discharged.
- V. To cool hot parts and add steam.
- VI. In an internally fired steam boiler, which may act as a starter.
- VII. As a jacket to hot parts, the steam used in a separate cylinder.

VIII. As an annular piston cooler to prevent contact of hot gases with working face of cylinder.

46. Methods of governing ;

I. Throttling air intake.

II. Throttling passage between the air and power cylinders.

III. Varying cut-off at power cylinder inlet.

IV. Varying combustion chamber pressure by fuel cut-off.

V. Varying internal pressure, by splitting the air from the compressor and sending more or less of it through or around the fire.

VI. Varying internal pressure by blow-off.

VII. Combination of varying initial pressure to power cylinder, and varying cut-off to same.

VIII. By throttling the exhaust.

47. Preheating and regeneration :

A heating of the air after compression, and before reaching the fire, will insure a good combustion, and if this is done by exhausted heat, it aids the economy of the engine.

I. Causing the air approaching the fire to first surround the fire-box as a jacket.

II. Causing exhaust and compressed air to pass on opposite sides of a plate.

III. Alternately sending fresh air and exhaust through the same chamber, which must then be present in duplicate.

IV. Combination of exhaust regenerator and fresh air fire-box jacket.

48. Fuel feed :

Coal may be fed by any automatic stoker, and from above, below, or the side ; we will omit these.

I. Oil by gravity, cut off by internal pressure.

II. Oil by pump, cut off by by-pass.

III. Oil by water displacement, cut off by internal pressure.

IV. Oil by injector suction.

V. Gas by pump.

49. Gas and oil burners :

I. Air and gas fed from opposite openings, jets impinging. Wilcox, Reeve.

II. Air and gas fed through separate openings, the jets mixing by impact with platinum rose. Villeneuve.

III. Air and gas mixed, fed through simple opening. Babcock.

IV. Air and gas mixed, fed through gauze screen. Brayton, Schumm.

V. Air and gas mixed by injector at burner, passing through openings into larger chamber, and products through openings similar to first, but larger in area. Schmid and Beckfeld.

VI. Volatile oil vaporized by heat of burner burning in atmosphere of air. Wilcox.

VII. Volatile oil vaporized by falling on hot plate in atmosphere of air. Schmid.

VIII. All kinds of oil sprayed into atmosphere of air. Nordberg and Shadall.

IX. Volatile oil dropped on grating, where it meets air, the mixture burning on one side. Brayton, Wilcox.

X. Hot-air atmosphere for an injection of fuel. Diesel.

50. Mixers and proportioners :

I. Mix at burner, proportions maintained only by pump of proper size.

II. Pass air through volatile oil kept at fixed temperature by exhaust.

III. Mix at burner, proportions maintained by pumps of proper size, aided by a device to feed both pumps at constant pressure.

IV. Proportions maintained by (a) movable partition between air and gas receivers, to keep pressures equal, and (b) proper double valve at the discharge.

V. Mix at injector, with no special device, for proportioning. Position of fire :

I. Directly under piston.

II. At side of cylinder; each end, when double acting.

III. In separate, continuously operated, highly heated chamber.

Cycles :

Nearly every cycle of operations can be followed. Engines that take air in one side of the cylinder and send it around to the other side, and those with two pistons moving in different directions, causing a change of position of the air only, will give a constant-volume combustion, and whether the engine follows, Cycle II., II A., B, or C will depend on the amount of expansion permitted. Engines feeding the cylinder with a flame may have a constant-temperature, or constant-pressure combustion, depending on the fuel used; hence III. or IV., with all their varia-

tions, are possible. Those engines that maintain a constant pressure in the combustion chamber will follow III., or some of its variations—which one, will be determined largely by the cut-off. A turbine system would follow III. most nearly.

51. Of the engines considered, and the much larger number not mentioned, not one, except the Diesel, is on the market to-day. This, with some, would be a sufficient argument to condemn the whole system, but a little study will show that the trouble is nearly always in the same place, and a little perseverance is all that is necessary to remedy the defect. Much more difficult problems than this have been solved successfully, but there was first necessary a recognition of the trouble and a good reason for spending time and money to overcome it.

The compressing of air is no new problem, and the using of hot gases is done every day in thousands of horse-power of explosion engines, so that these two parts of the engine may be considered solved, leaving, as the only doubtful essential, the fire, and here is the seat of most of the trouble.

52. The old engines, like Cayley's, using coal, were found to cut with ash, etc.; but, nevertheless, when everything was right, they ran and gave a good account of themselves. In the natural order of things, Brayton appeared with a gas machine. The air and gas were mixed in proper and explosive proportions, at the compression intake, and sent through a wire gauze grating to be burned in the cylinder. Clerk says of it:—"The engine worked well and smoothly; the action of the flame in the cylinder could not be distinguished from that of steam, it was much within control and produced diagrams similar to steam." The flame grating of gauze was the weak part, as an accidental piercing or overheating caused an immediate back flash and stopped the engine. Brayton could not stop this, so he tried a volatile oil with compressed pure air, but his burner was very crude and resulted in a goodly soot deposit. The case seemed hopeless, and doubly so when Otto appeared in the field with his successful engine, so Brayton gave up.

Here, however, was a working engine giving good results, both in economy and regulation, needing only a good burner to keep it going.

53. The immediate success and attractive principle of the simple one-cylinder Otto has held the attention of nearly all from then until to-day. One man there was who, not only did

turn aside, but, having turned, persevered, and he was rewarded by success—that was Diesel. He prepared an elaborate plan to imitate as nearly as possible the Carnot Cycle, with its isothermal combustion in a cylinder—certainly a striking novelty. But he did not follow it, as the low mean effective pressure of all the Cycles IV., which he attempted to follow, necessitated immense machines for the power produced; what he did do was to reproduce the Brayton engine with another burner and igniter. His hot compressed air did what Brayton could not do, but in everything else he was strictly Brayton with his Cycle III. of operations, which he ultimately followed. This is one solution, then, but not necessarily the best, as Diesel needs a very high compression to run, and, while this is the reason for his high efficiency, it makes a heavy machine. A little lower efficiency, with less weight, would be very acceptable, but this would preclude the Diesel burner.

54. A detailed study of the combustion of gas and oil should certainly lead to a still further opening up of this promising, though neglected, field of engineering. Produce a good fire and you must inevitably produce an improved Brayton engine, and this, in view of what has been said, is certainly a very desirable end.

While combustion, as a purely chemical process, has formed the subject of numerous papers and researches, leading to most interesting theoretical results of great value to physical chemists, not so many have resulted in the discovery of a new or useful mode of burning fuels.

55. Investigation long ago showed that the oils undergo a vaporization before combustion, and that the oil flame is really an oil vapor, or gas flame, so that a knowledge of the laws of combustion of gases will give us those of oil combustion, with the exception of the means of previously vaporizing the oil. In fact, the different methods of burning oil now in use vary chiefly in this second respect, the means provided for gasifying the oil. It would, therefore, be well to look at the question of gas combustion first.

Gases burn by combining chemically at high temperature with oxygen, and the study of their combustion may be most readily divided into classes whose characteristics are the ways in which this coming together of the gas and its oxygen are provided for.

56. This leads to the division:

Class I. Gas issuing from an orifice into a supporting atmosphere and where all the oxygen for combustion is derived from that atmosphere.

Class II. Gas mixed with oxygen insufficient in quantity for its combustion or for the formation of an explosive mixture, issuing into a supporting medium from which all necessary additional oxygen is derived.

Class III. Gas mixed with oxygen in quantities insufficient for complete combustion, but sufficient for the formation of an explosive mixture, issuing from an orifice into a supporting atmosphere, from which all necessary additional oxygen is to be derived.

Class IV. Gas mixed with oxygen in just sufficient quantities for combustion, issuing from an orifice into any sort of atmosphere. We shall call a mixture of this sort a "chemical" mixture.

Class V. Gas mixed with oxygen in such quantities as to form an explosive mixture, but with insufficient oxygen for complete combustion, burned in a mass by a single explosion.

Class VI. Gas mixed with oxygen in chemical proportions, burned by a single explosion in mass.

57. The first class of combustion is very imperfect, consequently only low temperatures result, while large excesses of oxygen over what is chemically necessary are required. It is to this very imperfection that we owe the efficiency of our ordinary gas jet as a source of light. The unequal distances travelled by molecules of gas before reaching the place where they can find and combine with the necessary oxygen, gives the flame a volume; *i.e.*, a certain portion of space is filled with the flame. In the study of combustion, as the origin of heat, this class is of no importance. Mixing the oxygen with the gas, previously to heating for ignition, as in Class II., is a direct aid to nature, eliminating the hunting process of Class I., or, at any rate, reducing it, and making necessary only the heating to the ignition temperature to cause combustion. This is shown in the immediate shortening of the flame over that of the previous class, and its loss of luminosity, while still retaining the volume character of the flame. It is the principle of the Bunsen burner, and the large class that follow it for use in furnaces, heaters, cooking stoves, and heating water in steam carriages.

58. In most of these the mixture of air with the fuel is made

by causing the jet of gas to impinge on a mass of air, some of which is carried along with the air under the double influence of gas friction and the heated top of the burner, whence the mixture issues.

Some other systems, of which the American Gas Furnace System is one, effect the mixture in closed chambers before exit at the burner.

Combustion of Class II. is characterized by the fact that there is an actual volume of flame; the flame is hotter than in Class I., which means that for a given flame volume, either more gas is burned, or the products of combustion are less diluted; the flame is less luminous and not of uniform color throughout its volume.

An infinite variety of details of arrangement in the exit and mixing of the air and gas may be devised with varying results for special cases, but we may say of all of them that though the combustion be very perfect and the amount of heat generated large, yet there is always a "flame volume," indicating a struggle, as it were, on the part of the gas and air in their final combustion. The combustion, though approaching perfection in many cases, is rendered so only by the use of a large excess of the oxygen chemically required giving oxidizing products of combustion.

59. It is only when we previously mix the gas and air completely and uniformly in the proper chemical proportions that we can get non-reducing, non-oxidizing products of combustion, and, since none of the heat goes to warm excesses of oxygen or of fuel, the temperature of these products must be the highest possible. Combustion of this sort is flameless, or, rather, what flame there is is without volume, having only length and breadth without thickness, and is, in fact, a surface.

Such combustion is governed by laws quite different from those under which the classes already noted operate, and it is to the combustion of chemical, and other explosive mixtures, that this section is mainly devoted.

60. Let us consider first the class mentioned as Class VI., in which a mass of chemical mixture—*i.e.*, gas and its needed oxygen—is confined in a chamber. If inflammation be provoked at any point of the mass, it will, by self-propagation, finally and successively inflame the whole mass. This is the first and fundamental principle of this sort of combustion. The investi-

gation of this propagation of inflammation by such men as Davy, Bunsen, Mallard and LeChatelier, Berthelot and others, has shown that:

(a) In any mixture, the rate of propagation is constant for a given temperature before inflammation.

(b) The rate of propagation for such mixtures varies with different combustibles, being, for example, very fast for hydrogen and slow for marsh gas.

(c) The rate of propagation increases with the temperature of the mixture before inflammation.

(d) The combustion is visible by reason of a flame-cap, or deep blue film of flame, which travels through the mass, and which, at any instant, completely separates all the burned from the unburned mixture.

This uniformity of velocity of inflammation would indicate that in a mass where inflammation had started at a point, the flame-cap, or surface of combustion, exists at any instant on the surface of a sphere whose radius is proportional to the time elapsed.

61. All this has been assumed to take place in a large mass of gas. If, however, the enclosing vessel be given special forms, certain other characteristics are brought out. One which is of interest to us is the fact that, when the enclosing vessel is a cylinder, or prism, in which the combustion surface travels with its centre on the axis, the velocity becomes affected by reduction of cross-section and that there will always exist for every such mixture an area of cross-section so small that the self-propagation ceases. This has been explained by saying that the walls carried off heat so fast that the small flame-cap could not generate heat enough to keep itself above the temperature of ignition. Davy secured the same effect by using his screen of wire gauze, which, if interposed in the path of the combustion surface, instantly cooled the same sufficiently to prevent the ignition of the mixture on the other side, provided, of course, the temperature of the gauze itself is sufficiently low.

62. When a neutral diluent gas, such as N or CO_2 , is added to a chemical mixture arranged for the above-discussed combustion, its effect is to reduce the rate of propagation, though not in conformity with any law yet discovered. Of course, there will be a point when so much of the neutral gas is present that combustion is impossible, but no reliable data are at hand

on this point, as the same conditions often give widely varying results.

While large quantities of a neutral gas may be added, without affecting the combustion except to decrease the rate of propagation, a dilution by a comparatively slight amount of oxygen will prevent it altogether. An excess of gas, it has been found, will act within certain limits like the presence of a neutral gas. By far larger amounts of fuel than of oxygen may be present in excess without arresting combustion.

63. This brings us to class V., where explosive mixtures are burned in mass, the mixtures having excess of fuel. The combustion is possible within quite wide limits, with no other effect than varying the rate of propagation. In fact, we see a great deal of it to-day in our gas engines. While, of course, we should, in these engines, invariably use the proper chemical mixture, they are seldom, if ever, constructed to maintain this properly, and, as a slight excess of oxygen will completely prevent inflammation, the error is always made on the other side; sooty exhausts bear testimony to this. The gas engine also gives evidence of the fact that neutral gases decrease the rate of propagation, for in some two-cycle engines which I have lately examined I find it impossible to get a vertical combustion line on the indicator diagram with a fixed ignition, except at very slow speeds—about 50 revolutions per minute. This is due entirely to the presence of exhaust gases in excessive quantities as diluents to the charge.

Some of the principles above noted as belonging to masses of mixture at rest will make clearer the nature of the problem of combustion of the same mixtures when in motion issuing from an orifice.

64. The desirability of being able to burn an explosive mixture continuously and non-explosively under commercial, rather than laboratory, conditions having long been obvious, a series of experiments was undertaken at Columbia University with this end in view. Many experiments were made and various results obtained, but as a full account would take too much space and avail little, only a few characteristic experiments will be noted as leading up to the result. Consider a mass of explosive mixture passing through a non-conducting tube with a uniform velocity v . Then, if inflammation be started at some point, the surface of combustion may remain at rest or move with or against the current. Denote the rate of propagation by r . Then, when

" $> v$ the surface of combustion will move with the current, and if the tube has an end, the flame will "blow off" and combustion cease; if $v = v$ the surface of combustion will remain at rest, other influences being inoperative; if $v < v$, the surface of combustion will move back toward the source or "back flash."

Of course, a small tube of heat-conducting material will exert considerable cooling effect, but for the present we will not consider such tubes.

In a practicable system of burning an explosive mixture continuously, we may state the following as desiderata and later see how they can be secured.

I. "Back flashing" must be prevented.

II. "Blow off" must be prevented.

III. Combustion surface must be localized.

IV. It must remain localized for wide ranges of feed or velocity of flow of the mixture.

V. The localization must be unaffected by changes of temperature.

VI. Large or small quantities must be burned without affecting the above, and the transition from very small quantities to very large, or *vice versa*, however sudden, should be easy.

65. The first requirement might be accomplished in three ways:

(a) By using a long tube of some conducting material and so small in diameter as to prevent the passage of the flame-cap under any circumstances.

(b) By using wire gauze screens.

(c) By causing the mixture to flow at some point with a velocity always greater than the rate of propagation.

The first (a) is impracticable, as it permits of only small quantities being burned; the second (b) will not work when the wire gauze gets hot; this leaves (c), which is practicable, as a valve in a pipe will answer for the necessary contraction and consequent increase of velocity. Hence we must put down as the first requirement in our desired method of combustion the following. At some point before the combustion surface is reached the velocity of feed must be such that $v > v$.

66. Requirement II. might be accomplished in three ways:

(a) By so reducing the velocity after passing the high-speed point that we have at some surface $v = v$.

(b) By suddenly increasing the temperature of the mixture so

as to increase the rate of propagation while v remains constant; or,

(c) By both reducing v , by spreading the current, and increasing r by heating.

All of these ways are practicable; but, as a reduction of velocity alone, or a sufficient heating alone would not produce the desired results so well as both operating together, there was introduced as the second requirement in our desired method, the following. After passing the point where $v > r$, the velocity of the mixture should be so reduced and its temperature increased as to make $v^1 = r^1$.

67. With these conditions in mind, let us consider an experiment. Let the mixture issue from an orifice into the air. By properly regulating the velocity of exit, the flame-cap can be maintained at the orifice—the only device with which I succeeded in this experiment was by causing water to drip into the supply chamber; the position of the flame-cap is so extremely sensitive to changes of flow that all other methods which were tried for obtaining a constant velocity of exit, variable at will, failed—increase the flow slightly, and the flame-cap will lift off. This may be done until the flame-cap is as much as 2 inches (with illuminating gas and air) from the orifice before extinction takes place. It would seem that the impinging of the jet on the atmosphere should spread it and so reduce its velocity, but no appreciable increase of diameter could be observed. When the cap is close to the orifice, it is of a deep blue color, uniform in shade over the disk, and the edges are sharply defined; whereas, as it lifts off some distance, it becomes indistinct and unsteady at the edges until, at the moment of extinction, it fades into nothing. When the cap is away from the orifice, while there is no visible connection with the source of supply, there really exists a column of mixture extending from the orifice to the cap and passing through the atmosphere. Naturally, at the surface of this column, diffusion will take place, and the longer the column, the greater will be this diffusion effect, thus affecting the composition of the advancing column of mixture and causing partial loss of gas. This is the real cause of extinction.

68. From these experiments we can draw the conclusion that the current cannot be sufficiently reduced in velocity by issuing into an atmosphere of lower pressure to prevent “blow-off” before diffusion with the atmosphere so alters the character of

the mixture as to cause extinction before reaching the surface of combustion. This calls for a new condition besides those noted in the requirements for combustion. The reduction of velocity of the mixture, after passing the place where $v > r$, must be accomplished in such a way as to prevent diffusion with any other gas.

69. To prevent this diffusion, there naturally suggests itself the expedient of surrounding the issuing jet with a shield of larger diameter, to still permit of the desired expansion. This is shown in Fig. 47, and is essentially the same as proposed by Ladd, Schmid, Beckfeld, and others. The mixture must issue from orifice a with a velocity $v_a > r$; this will prevent "back flash." If the distance from a to b is long enough to allow the gas to spread and reduce velocity, "blow-off" will not occur until $v_b > r$, and within these limits the flame-cap should re-

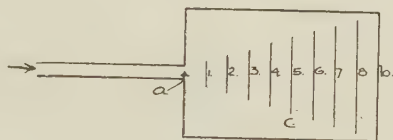


FIG. 47.

main within the shield. A trial shows that when $(Dia)_b$ is but slightly larger than $(Dia)_a$, the feed velocity may be varied in about the proportions noted, but this means that we are confined within very narrow working limits. The flame-cap seems to lose its flat, volumeless character for some reason not at first clear. When $(Dia)_b$ is much larger, say four or five times $(Dia)_a$, a slow increase of feed velocity above r reveals the advancing flame-cap just as if the shield were not there. Later, a slight spreading is noted, and then the flame actually begins to show volume, as if there were no longer an explosive mixture present; this heats up the shield. A little consideration will show this to be due to the diffusion of the advancing and slightly spreading column with the products of combustion within the shield, and the high temperature of the shield helps to maintain a combustion of what is now a diluted explosive mixture beyond a point where that combustion would be possible if cold. An increase of velocity will cause extinction by "blow-off."

70. Here the results are somewhat better than in the last case without the shield. The principles operating, with the results are: back flash prevented by sufficiently great initial velocity at a ; a spreading to reduce velocity, but very slight and insufficient, as proved by the narrow working limits; diffusion is not prevented; gas is partly heated before burning by the shield, which helps to continue combustion. If the advancing column did increase in cross-section and decrease in velocity while advancing, successive possible positions of the flame-cap would be as shown at 1, 2, 3, 4, etc., of Fig. 47.

It is obvious that at any point between a and 7, such as 4, the cap is surrounded by products of combustion, and the advancing column of mixture is passing through an atmosphere chiefly composed of the same, resulting in disastrous diffusion.

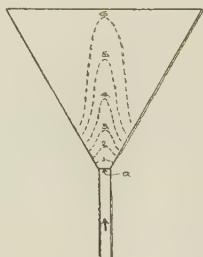


FIG. 48.

This at once suggests giving the shielding envelope the form of a cone, supposing the orifice circular, so that the flame-cap at any instant may entirely fill up the space between the walls.

71. Apparatus with this end in view was tried and gave some interesting results. Fig. 48 shows a cone of 45 degrees angle, with a $\frac{1}{2}$ -inch orifice such as was used. The velocity of feed was so adjusted as to cause the flame-cap to advance slowly from a , with the expectation stated above. The flame-caps at successive positions took the forms shown at the lines 1, 2, 3, 4, 5, 6, etc., and finally "blow-off" occurred. Since the only place where the combustion surface can remain at rest is on a surface where $v = r$, and since, secondly, r is here constant, the curves indicating the intersection of the combustion surfaces by meridian planes, give us graphical values of the velocity of the advancing column of mixture. It is seen that the expected spreading did not take place, and that at any circular cross-section of the

cone, the velocity was greatest at the centre, decreasing toward the edges.

The curves 1, 2, 3, etc., are really cross-sections of successive constant velocity surfaces in the advancing column, and the surface of combustion will lie on that surface of constant-transmission velocity where $v = r$.

72. A constant-velocity surface may be defined as a surface at every point of which the moving particles of gas have equal instantaneous velocities. If these successive surfaces had remained flat or nearly so, the proper sort of spreading of current and uniform decrease of velocity would be indicated. This gives us an accurate definition of how we want our velocity reduced after passing the point where $v > r$. The velocity of the advancing mixture must be reduced without diffusion, so as to keep the surfaces of constant velocity of such form that adjacent points on any one will be at approximately the same distance from the point where spreading begins. Reducing the angle of the cone, while helping matters considerably, reduces the range of feed velocities within impracticable limits.

73. Many ways of bringing about the above were tried, but only one seemed preëminently good both by reason of its simplicity and effectiveness, for it fulfils almost perfectly the requirements proposed for our desired method; this is, to fill the cone with fragments of refractory material such as pottery, broken crucibles, bits of magnesite, or any other rock that will stand the high temperature without fusing. In cones of 60 degrees, and with a $\frac{1}{4}$ -inch orifice, I have found pieces about $\frac{3}{8}$ -inch diameter to answer well.

These separate pieces of solid matter interpose many reflecting surfaces without materially hindering the advance of the mixture, and cause it to spread in the way desired, keeping the surface of combustion spherical and preventing diffusion. A variation of velocity causes the spherical surface of combustion to vary only in diameter, and the limits of feed are determined only by the size of the cone.

74. A cone of given altitude will give the greatest range of variation of diameter of cross-section when its angle is 180 degrees. This is a plane surface which, with the orifice and broken rock, should appear as in Fig. 49. Here the surface of combustion is approximately a semi-sphere. Trial shows that this arrangement works perfectly, and the limits of feed are deter-

mined only by the size of the pile of rock surrounding the opening. A cone of 360 degrees, or no cone at all, suggests the surrounding of the nozzle by broken rock without any enclosing walls (Fig. 50). This arrangement also works remarkably well.

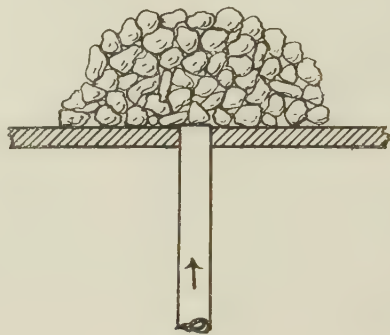


FIG. 49.

The surface of combustion is here approximately a sphere, giving the greatest possible increase in area of the surface of combustion for the distance travelled from the nozzle.

If d denote the distance from the point where spreading begins to the surface of combustion and S the area of the surface, we have :

For a cone, $S = \pi d^2 \tan^2 \alpha$.

For no walls (Fig. 50), $S^1 = 4\pi d^2$.

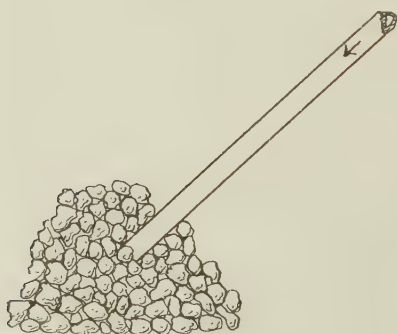


FIG. 50.

75. Not only is the greatest possible range of action by velocity reduction thus obtained, enabling the greatest possible amount of mixture to be burned in a given volume, but this

amount is further augmented by reason of the increase of the rate of propagation caused by the passage of the mixture between the hot fragments. Hence both principles operate simultaneously toward the desired end.

We have thus arrived at a method of continuously burning explosive mixtures of all sorts, whether in the chemical proportion or not, as classified in IV. and V.

76. The method fulfils all the conditions set down as necessary, and may be stated as follows :

I. Cause the mixture to pass a point where its velocity of transmission shall be always greater than the rate of propagation of inflammation through the mixture. This may be done by a valve in the feed-pipe.

II. So spread the current of mixture after it passes this point of high velocity that surfaces of constant-transmission velocity shall be of such form as to keep adjacent points on any one at approximately the same distance from the point where spreading begins. The whole spreading must take place so that the advancing unburned mixture cannot diffuse with any other gas. This can be accomplished by surrounding the orifice with solid fragments, introducing numerous reflecting surfaces which accomplish the spreading; also, by the passage through the interstices between this solid matter, the mixture is heated and the rate of propagation increased, making possible the burning of more mixture in unit volume.

77. When a chemical proportion is maintained in the mixture, all the combustion takes place on the *combustion surface*, giving absolutely neutral products of combustion; but when an excess of gas is present within certain limits, all gas that can find oxygen burns explosively between the solids, while the excess acts merely as a neutral diluent to be burned when it meets an oxygen atmosphere later on. By properly placing the oxygen atmosphere to burn the excess gas, we can get the hot products either *reducing* or *oxidizing*—reducing after leaving the explosive-combustion surface and before meeting the excess of oxygen in the atmosphere, oxidizing after that meeting.

It might be here noted that the principle well known in explosive combustion at constant volume, and constantly operating in the gas engine, that “to a chemical mixture of air and gas there may be added large quantities of gas without altering the explosive properties of the mixture,” is, by these experi-

ments, extended. It appears that in explosive combustion at constant pressure, or, as I have called it, "continuous combustion of explosive mixtures," the same principle applies, and, though no real proportion measurements have yet been made, it seems to a wider degree. That is to say, that in the method here described, mixtures of air and gas, with gas in excess of the amount the air present can support, will burn explosively. The excess gas present acts merely as a neutral diluent, such as the nitrogen of the air. It is a fact also that, as the solid fragments heat up, the excess may be greater than when they are cold.

78. Another interesting thing noted in these experiments is that an explosive fire will sometimes emit a musical note; it may be that this is always true and that its absence at any time is due to lack of the proper resonator. This would seem to indicate that what to the eye appears as continuous combustion, is only approaching the limit, which is continuity, and that in reality single explosions in rapid and *regular* succession are taking place. It would be interesting to determine whether the temperature or kind of gas has any influence on this note.

79. The perfection of the gas combustion above discussed and the simplicity of the apparatus make the method highly satisfactory, and the solution of the difficult problem of explosive-gas combustion lends encouragement to the even more difficult case of oil combustion. The experiments with oil, though not yet complete, promise to give equally satisfactory results; in fact, it is almost certain that they will. However, the oil system has so far been tried in only a few cases, and it is not wise to announce the complete success of the system until all possible conditions have been met.

80. It was shown that there were only two classes of combustion worthy of consideration for use in internal-combustion engines, and only two cycles that promised returns commensurate with the labor and time that might be expended in their development--the Otto and the Brayton. The Otto is simple to carry out in practice, and is now, to all intents and purposes, fully developed, while the Brayton has hitherto failed, chiefly because of the difficulty of handling explosive mixtures in the desired way. This difficulty now removed, puts the Brayton cycle on a different basis, making the system quite as feasible as the Otto, and, in most respects, promising better results. Not

only this, but the fact that the oil combustion will almost certainly be put within as easy reach, adds another point in favor of the Brayton cycle, in the carrying-out of which any sort of oil may be used, whereas the Otto is here barred.

It is not necessary to enumerate here the comparative merits of the two systems, for that can be easily judged by what has already been stated.

81. There is one point, however, that should receive notice, that is, should we operate Brayton cycles with intermittent or continuous combustion? With intermittent combustion the fire burns within the cylinder, and as nothing but fuel and air pass the inlet valves, they can be the more easily kept cool; while, on the other hand, the placing of the burner beyond the valve presents two undesirable features: first, the clearance must be unusually large; and second, the intermittent feed and cut-off of air and fuel at just the right time, without alteration of proportion in a fraction of a second, introduces a condition very difficult to meet. Continuous combustion within a fire-box is easier to handle, there being no alterations of feed and the clearance may be as small as we please, whereas we have as undesirable the feeding of hot gases past the inlet valves.

Which of these alternatives will prove the better for use, in the system of engines under treatment, can be decided only by actual construction, but as either will work, there is no great risk involved in building.

The paper on liquid fuel combustion which follows belongs properly in the body of the previous paper and should follow other matter of paragraph 79 page 60,* but at the time this was written the work on oil had not yet been completed. The slight lack of continuity also which is apparent as well as some repetition is due to the way in which it was found necessary to present this somewhat extended work, *i. e.* the preparation of separate papers dealing each with one phase of the work but each sufficiently complete within itself to make good reading and hold the interest to but one topic. With this in mind it is believed that the connection and interrelation existing will not seem too strained.

* A. S. M. E., Dec., 1901.

NOTE.—This paper is sent to you that you may examine it in advance of the meeting, and prepare any discussion of it which you may wish to present.

It is issued to the membership in confidence, and with the distinct understanding that it is not to be given to the press or to the public until after it has been presented at the meeting.

The Society as a body is not responsible for the statements of fact or opinion advanced in papers or discussion. (Art. 44 of its Rules.)

As there will be no adequate supply of extra copies, and papers are liable to be read by abstract only, preserve this copy for your use, and

BRING THIS COPY WITH YOU TO THE MEETING.

(Subject to Revision.)

No. 0934.*

LIQUID FUEL COMBUSTION.

BY CHARLES E. LUCKE, NEW YORK.

(Non-Member.)

PRESENTED BY R. H. FERNALD, NEW YORK.

(Associate Member of the Society.)

1. OIL combustion, considered as a rather complicated series of physical actions, has never received the attention due to its importance. There have appeared from time to time men who, taking up the corresponding question for gases, gave to the world a series of researches which leave but little to be desired, and the very perfection and elasticity of our methods of burning gases brings into stronger relief the narrow limits of present practice in oil combustion. Before we can hope to design special and proper furnaces this problem must be attacked from this standpoint, and the physical operations will when brought together and classified give us the principles of oil combustion. A detailed and minute treatment of this question would call for a lifetime of study, but some of the principles are more prominent and appear more evident than the others; a few of these have appeared in the course of some experiments undertaken for an object noted later.

2. The analytical treatment of the combustion of gases greatly simplifies the problem of oil combustion. By classifying the gas-burning methods according to the mode of bringing the air and gas together, it was found that there were, broadly, two great

* To be presented at the Boston meeting (May, 1902) of the American Society of Mechanical Engineers, and forming part of Volume XXIII. of the *Transactions*.

divisions of all systems, those in which a supporting atmosphere was necessary, and those in which, because of the self-propagation or explosive property of the burning mass, no supporting atmosphere was necessary. Moreover a distinctly different set of laws of physical action holds in each case. The laws of combustion for explosive mixtures with their volumeless flames are radically different from those for all other mixtures the combustion of which calls for a supporting atmosphere, giving rise to a volume of flame due to the meeting of the fuel and oxygen at different points, and at each springing into flame when juncture is effected. The volumeless flame of the true explosive fire depends for its localization and maintenance on the relation between the rate of propagation of inflammation in the mixtures and the velocity of translation, together with the extent of freedom from diffusion of the fresh mixture with the products while approaching the combustion surface.

3. For a comprehension of the different cases of oil burning, we must add to our knowledge of gas combustion something on the vaporization of the oils, since it is conceded that oil will not burn as such a distillation or vaporization preceding the actual combination with oxygen. So that different oil systems will differ chiefly in the method of producing the oil vapor, and in the method of causing a meeting of this vapor with the air. Any two systems which agree in these two points must be brought together as coming under one class, but perhaps differing in details which may or may not be essential to good results.

4. Of all the different systems proposed we can, according to the above, note only three different general classes :

I. The burning of oil in an atmosphere without previous treatment by air or heat. This class burns the oil (*a*) from a surface either that of the liquid mass or that of films artificially produced by sand, stones, fibrous or metal wicks. The vapor burns immediately as formed, and hence there can be no mixing with air, the flame existing merely in an atmosphere which may be often renewed or not, *i.e.*, depend on blowers or mere convection. The fires resulting from this class are grouped for action and effects with the first kind of gas combustion, a jet of gas issuing unmixed into an atmosphere of air.

There may also be included (*b*) those retort or (*c*) spray vapor producers which deliver oil gas, as just noted.

Oil burning by methods coming under this class must be subject

to the same merits and defects as the gas combustion noted, however diverse, complicated, or ingenious the details of operation or construction may be.

II. Under this class will be grouped all oil fires capable of producing what is known in gas combustion as the "Bunsen effects." That is to say, the oil is vaporized in such a way as to permit the mixture with it of a certain amount of air before it reaches the existing flame, and having reached the existing flame requires an oxygen atmosphere to support combustion.

Any system producing vapor which can be handled as can a gas may also be included.

III. The third class of oil combustion will include all those methods in which the oil is vaporized and mixed with air in such proportions and in such manner that there will result an explosive mixture of oil vapor and air. Such oil fires will be subject to the laws of combustion of explosive mixtures. The vapor may be produced in retorts by boiling a mass of liquid, or by spraying oil on hot surfaces and then conducting it to a point where it may mix with air, or the oil may vaporize by contact with or approach to a hot surface in the presence of the air.

5. The most natural and earliest practical method of oil burning was that of simply lighting the surface of a mass resting in a pan. The amount of heat that could be developed depending on the surface of oil exposed led to a spreading of the oil over plates and running over numerous grooves and in the formation of cascades, etc. The high flash point of some oils, *i.e.*, the high temperature at which they give off combustible vapor and the presence of the liquid mass made it impossible to burn them in this way, and hence was brought about one of the first improvements in oil combustion. The wick system results from a desire to produce more vapor, and this from oils of high flash point; by it oil is caused to spread out over a large surface in as thin a film as possible, and is then subjected to heat. Being in a thin film it is easily vaporized because of the small quantity at any point and the ease with which the substance supporting the film can be heated and kept hot, the vapor thus produced burns as it appears in an air atmosphere.

When the film bearer has a low specific head the vaporization is the more rapid at the surface but slower beyond; with a metal film bearer the conduction of heat beyond the surface causes a vaporization at more points and insures more complete vaporiza-

tion by a superheating at the surface; the superheating may even decompose the vapor.

6. These wick burners are easily illustrated by a pile of sand, fragments of brick or fibrous material in a pan of oil; wire net may also be substituted for the fibrous or other material.

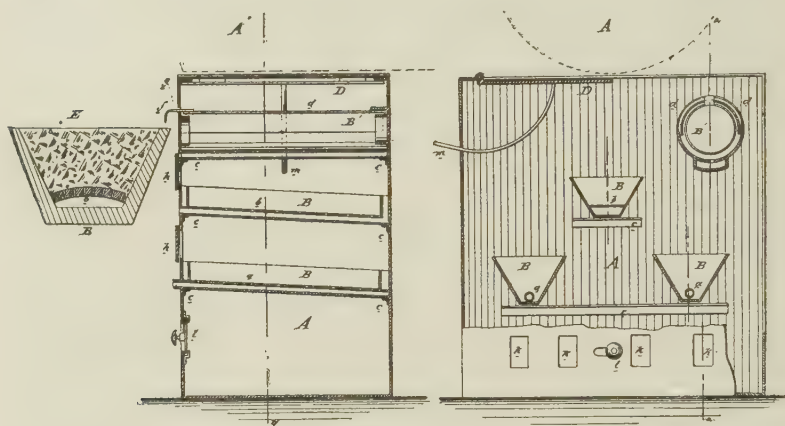


FIG. 1.

For these burners to work at all the surface, at least, must be hot, and when acting in an atmosphere the latent heat of evaporation of the oil will tend to keep the temperature down, resulting in steady conditions. There will always be a limit of rapidity in such combustion, since a constant state will be reached for the

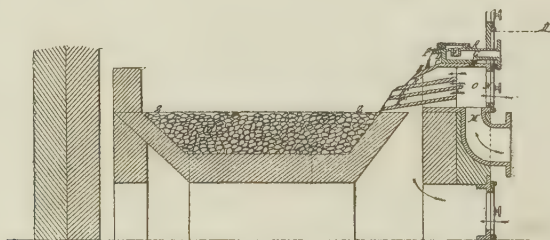


FIG. 2.

wick temperature and rate of evaporation, and, consequently, for the combustion, making regulation difficult.

7. Such systems must necessarily be slow heat producers; however perfect the combustion and disadvantages of the first class of gas combustion, we must add a few more characteristic of the methods of evaporation.

Fig. 1 shows the simple pan-wick system of Weeks, and Fig. 2

the cascade of Verstract, which is combined with a wick at the bottom to burn what escapes from the cascade. This is selected for illustration because it is also an example of an attempt to produce Bunsen effects in the mixing of air with the vapor. It does not succeed in this, however, because when the liquid surface is present the flame will locate there, and the air blown through the slits *R* on the falling oil sheet can only have the effect of an often renewed atmosphere, *i.e.*, a wind; no mixing of vapor and air is possible.

8. However, the surface of vaporization is increased, hence more vapor is produced, and, in addition, the air blown through helps to accelerate the combustion; but in spite of this the action is precisely the same as before, a flame of oil vapor burning in an

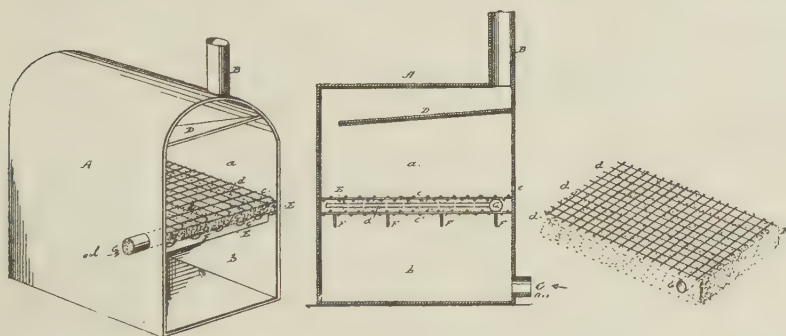


FIG. 3.

atmosphere of air. The improvement then is not one of class but of detail.

Improvements of the same sort on the wick method, aiming to lift the wick from Class I. to Class II., and get Bunsen effects result in the same way. Air blown through the wick chills it and retards vaporization, in addition to slightly lifting the hot part of the surface-flame farther from the vaporization surface, which should be kept hot. An illustration of this, Fig. 3, is the method of Hubbard. A mat is provided with a pipe system to deliver oil at numerous outlets in the mass, with the intention of saturating the mass. Then air is blown through the mat. The intention is to vaporize the oil in the mat by the heat from above, and the vapor, mixing with the air passing through, is to ignite on the top. It will be readily seen that as each outlet is a source of oil feed to the mat, we have a large number of wicks grouped with air blowing on them.

9. Supposing a vaporization to take place immediately on issuing, as is expected, and which fact, of course, depends largely on the fuel used, we will have each nozzle a source of gas, and there will result a number of gas jets blowing into the mat. The ascending air current will lift the gas jet, and there will result practically a cone of gas with apex at the orifice, surrounded by air. At the edges there would be some diffusion, and beyond the cone a moving atmosphere of air. If the mat were thick enough and the air current not too swift, there might result an approach to a Bunsen flame in an atmosphere of air within the mat. If the mat were not thick enough, and the air current moved too fast, there would be at the surface a Bunsen effect. In no case could there be an explosive effect of Class III., because of the relation of air and oil vapor supply, the one surrounding the other, making at every point a constantly changing proportion. Were the air and oil vapor discharged into the mat through the same orifice the effect would be quite different, as will be seen later.

10. Air blown on the top of a wick makes the flame a little more vigorous only because it renews the atmosphere instead of depending on convection, but the process would not change the combustion otherwise. The two systems of surface evaporation from the liquid mass and from the wick film both demand, in order that the action may be continuous and non-clogging, an easily vaporizable oil that will not distil into parts of different vaporization temperatures. With such an oil obtainable, of course the next obvious step is to simply boil it in a retort, producing vapor which can be used exactly as gas and by all the means known for gases. However, an additional precaution must be taken, that of preventing decomposition of the vapor produced by undue heating before burning.

This would be a great advance over the methods noted before, given only the proper fuel, and we can produce any sort of fire from the illuminating flame, through Bunsen and blow-pipe effects to the more recent explosive fires with their high temperature and rate of consumption, and with each obtain perfect regulation.

11. To vaporize oil in retorts requires that

(a) The oil be not subject to fractional distillation, but that all of it must vaporize at the same temperature for any given pressure.

(b) The temperature shall not rise above this vaporization tem-

perature or decomposition of vapor will result with deposit of carbon to choke the passages.

(c) The vapor once produced must be prevented not only from superheating before reaching the fire, but also from condensing.

These conditions are exceedingly difficult to get, and no oil cheap enough to be used for fuel in competition with coal is available for the system which is otherwise very attractive in its simplicity and range of possible effects.

12. These oil vapor producers may be operated by (a) the boiling of a mass of oil, (b) the vaporization of a spray, stream, or drops by contact with hot parts, and (c) by the carburated air method. The first two, so far as their resultant effects go may be grouped together, but the third has been found of value in many cases where the selection of the required fuel is not prohibited. Air is brought in contact with liquid surfaces, and passing off carries some vapor with it. We have here a mixture of air and vapor burnt in the atmosphere of air, or we may go farther and form the explosive gas requiring no atmosphere to burn. In just what proportions of air and vapor the mixture will be delivered from the carburettor depends on the temperature of the air, the intimacy and length of time of contact with the liquid, and the temperature at the evaporation surface. Of course, the temperatures of the carburettor will tend through evaporation to become continually lower, and this must be guarded against.

13. These oil vapor systems differ but little from the pure gas systems, and whatever can be done with gas can be done with these vapors, giving fires of classes I., II., and III., provided the proper fuel is available, and, if the necessity for the fire is so urgent that cost is not the most important consideration, they may be very useful.

We have not yet noted, however, any system which will enable us to burn heavy oils, or those which fractionally distil, leaving a residue and of these petroleums and some of the petroleum products form the largest and cheapest source of liquid fuel supply.

With the spray or atomizing system we have something radically different from these so far considered, inasmuch as any kind of oil may be used and a good fire obtained with each. Here the oil passes through a small opening, where it meets air issuing at a high velocity and is by it thrown into the firebox as spray. The firebox being filled with flame and lined with brick also quiet

hot, each particle of oil is vaporized in the presence of air, and the products of combustion of previously consumed oil particles.

14. The temperature of the fire resulting is extremely high, and this led to the use of steam for the spraying agent, the injecting nozzle having other openings through which air passes under the influence of the chimney draught and partial vacuum produced by the jet.

The action here is probably more nearly explosive than anything else. It was noted that the rate of propagation of combustion in explosive mixtures is very much increased by high temperatures. When an explosive mixture is forced into an enlarged chamber previously made very hot, blow-off is prevented for quite a range by this increase in the rate of propagation. The oils commonly used in the spray have a very high temperature of vaporization, and it is more than probable that, moving with the air in the hot parts of the firebox, at the high temperature of the mixture when vaporization takes place, the rate of propagation becomes so high that blow-off does not occur. However the action is not the best even though explosive, for there is a large admixture of products with the jet, particularly at the edge and at reflecting surfaces.

15. The entering jet, approximately conical in form, is composed of a large number of oil particles, each surrounded by some air and some steam. As the jet approaches the hot section, the oil springs into gas and the gas with the air into flame, the steam is inactive until very high temperatures are reached, when it decomposes and acts as a cooler. The vaporization of the oil is accomplished either in space while the oil particle is in motion surrounded by air, or by contact with some of the solid surfaces of which a good many are provided in the form of arches, bridges, baffles, etc. All that can be seen is an orange glow and the course of the jet is invisible, except near the nozzle.

The system requires that the spray be formed, and for this air or steam under sufficient pressure must be provided, numbers of baffles and bridges to break the current after it has entered, in order to scatter the jet and distribute the heat; a firebox of sufficient capacity to allow the formation and vaporization of spray and its final combustion; small openings at the nozzle.

16. Many auxiliaries to the spray have been used, but of these the most notable is that of Kermode, who sprays with heated air directed upon a bed of bricks or asbestos placed on the fire area.

He provided this loose fire-bar covering simply to cover the bars easily but noted that by their presence the action was improved, for a reason which will be seen later.

While most of these spray systems depend on a pressure drop of air or steam to atomize the oil and these seem to have given the best satisfaction, yet there are some others which work on the few ounces pressure of a fan. The oil is conducted to sharp points by capillary action and blown from them by the blast; tests of these generally show higher oil consumption than the compressed-air system, probably because of the lower temperature, resulting from more air and slower burning.

17. The subject of oil combustion in general is very interesting to the laboratory experimenter, and as a system was desired which would burn enclosed under pressure for use in the internal combustion-engine, a series of experiments was undertaken at Columbia University to find, if possible, a method which was adapted to these conditions.

With the knowledge of what had been done with oil fires in other applications as a guide, the first series had for its object the determination of the principles that should govern enclosed pressure fire systems. These principles once determined, it was hoped that the desired method would appear. Some of the experiments made together with the deductions therefrom are here briefly presented for a record, as they may be of value to other workers in the field. Engines which work by passing air through a fire and thus expanding the volume at constant pressure, impose on the fire some conditions not easy to satisfy.

18. Air must be compressed into the firebox, and at each delivery of the compressor there will be a pressure increase on the fire; similarly at each admission to the engine cylinder there will be a pressure drop, and while we may call the system a constant pressure combustion system, this cannot be strictly true. What is constant is the mean pressure, and even this may vary after a limited time, for variation of admission and cut-off will change it. So that a fire to work successfully in this apparatus must be unaffected by pressure variation whatever may be its extent or suddenness.

One of the greatest advantages that may be derived from this type of engine over the explosive, for example, is the possibility of employing the cheap and safe heavy oils. But to realize this advantage we must add to our conditions one imposing the require-

ment that heavy oils shall be burnt. And finally, the products of combustion must be delivered at a constant temperature, and that as high as possible. Moreover, this maximum must be known to the designer who proportions his cylinders and mechanism for some particular volume expansion dependant on this maximum.

19. The most radical difference between these conditions and those imposed on an ordinary fire is that of burning, enclosed under a pressure which may vary widely and suddenly; so it was decided to first try to obtain a fire which would do this regardless of the fuel used or the delivery temperature, and this being attained to experiment with the other conditions by making appropriate modifications.

20. One burner which seemed to give a good steady Bunsen effect in ordinary use was that of the gasolene or kerosene

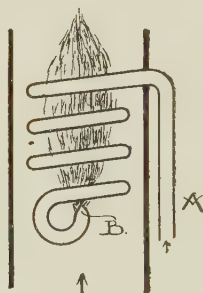


FIG. 4.

soldering, torch, or cook stove. This seemed so simple and unlikely to be affected by pressures that the principle involved was that first tried, oil fed through a self-vaporizing apparatus and escaping as gas.

Kerosene was fed through a coil of small brass tubing as shown in Fig. 4, the oil flowing from the top toward the bottom burning at *B* and playing on the coil. It was expected by a long coil to obtain a perfect vaporization. This device was found exceedingly irregular in action, no matter how carefully the feed was adjusted, the vapor delivery was never steady, varying from a long flame to total extinction. Matters were somewhat improved by enclosing the coil in a shell insuring a uniform heating throughout. After working for some time in this way the operation stopped, and the tube was found full of solid carbon at the lower part, showing a decomposition of vapor in that part. Gasolene, alcohol, etc., could be used, but not petroleum and heavy oils.

21. There were two faults prominent in this arrangement: first, the down-feed through a variously heated coil, gave rise to uneven vapor generation; second, the passage of the formed vapor through the heated part where decomposition could occur.

In the next burner it was intended to do away with both of these faults, the first to be eliminated by having a large mass of liquid boiling, and delivering vapor in such a way as to avoid superheating and so eliminate the second fault.

The apparatus of Fig. 5 was made. Oil enters at *A*, is dropped to *X*, where it boils in the chamber, being heated from below; the vapor generated passes around *BC*, feeding the flame. *B* is a valve which permits shutting off vapor delivery, and by the increase of pressure also the feed which was under constant head.

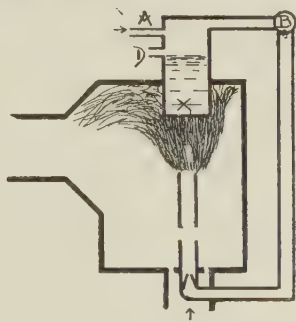


FIG. 5.

Any rise of oil level was prevented by the overflow *D*. Air entering at the bottom with the vapor a very good Bunsen effect could be produced when burning free and with naphtha as fuel. When enclosed, however, and with pressure put upon the chamber the flame became very irregular, and any quick change of pressure always resulted in extinction. With kerosene there was considerable vapor condensation in the drip. Various modifications of these vapor generating pressure oil burners were tried, but all were unsatisfactory for enclosed pressure use. The boiling oil generates within its chamber varying pressure depending on the rate of boiling, and rate of efflux of the vapor. The rate of boiling or vapor generation, if the flame below is kept constant, will depend on the pressure on the surface of the liquid.

22. This, in turn, depends upon the pressure on the flame and the size of opening in the vapor pipe. We thus have a number of conditions surrounding the vapor supply, from which the air

supply is free, but the air supply has varying conditions of its own, and as these double conditions are never, as it were, in phase, the result is failure. The only way in which we can keep the proportions of air and vapor right is by observing the flame, and this is, of course, out of the question when it is enclosed. When conditions can be kept right, a very good fire can be made with this burner, using kerosene, gasolene, naphtha, alcohol, etc.

Some other experiments leading from this brought out the fact that much better results could be obtained if the boiling is confined to the surface of the liquid rather than allowed to exist throughout the whole mass. To get this result a pipe was bent, as shown, Fig. 6, and oil fed from below to the enclosed length, which becomes hot on top from the oil vapor jet *B*. With a constant head, a flame could be kept lighted under pressure, and enclosed up to the feed-head. A sudden change, however, created

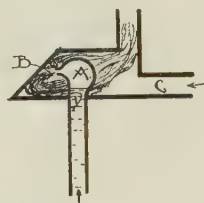


FIG. 6.

trouble. The vapor delivery depends on the difference in pressure between the chamber and the feed-head, and the flame grows smaller, allowing the hot plate to cool when it should be getting hotter. The proportions could not be maintained at all constant under variable pressure, though the burner would work all right when proper adjustments could be made.

23. With a feed varying with the chamber pressure a slight improvement resulted, though even then the result was not satisfactory. There was carbonization at the orifice with kerosene, and the apparatus would not work at all with heavy oil. Sudden pressure changes invariably caused extinction. The amount which can be fed economically can be varied but little, and not so to keep any constancy of proportions with the air.

To maintain some such constancy of proportion was necessary, because the ultimate object of the oil fire was to heat the air, and different quantities of oil burnt in the same air will give different temperatures; and if the proportion cannot be predicted certainly

the final temperatures cannot, and the fire is useless for the purpose in hand. With the purpose of keeping some sort of ratio between fuel and air, an air suction oil lift was tried, Fig. 7.

24. It was not intended that the complicated action of the common atomizing spray should result, but only that the air should lift oil in quantity somewhat in proportion to its own quantity. This oil is blown with some air through a flattened attenuated opening *A*, where it is spread out without changing its velocity,

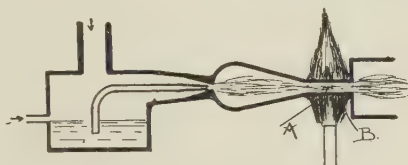


FIG. 7.

and then brought in contact with an externally heated plate, *B*, to be vaporized, the action being similar in effect to that of the carburettors used in Priestman oil engines. It was found that there was one rate of air feed at which just the right amount of oil would lift, a variation either way changing the action materially. When enclosed the slightest change of pressure results in bad action, sooting, flooding, and extinction. A number of similar injector oil lifts were made, and the conclusion reached was that none could be depended upon to produce the action desired. To further test the principle of carrying oil by the moving current

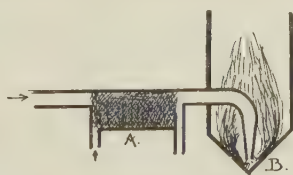


FIG. 8.

of air either as mist or vapor, the arrangement of Fig. 8 was tried. Here an irregular mass of wire net fills the chamber *A*, which is about one-half full of oil. The wire threads draw up by capillary action the oil from the surface, spreading all over the wires and some of the spaces between, making conditions very favorable for the air to take up either mist or vapor as the case may be. The issuing jet is reflected back upon itself and heats the nozzle, insuring that any mist shall become vapor.

25. The opening need not be small. It worked very well for kerosene and better for gasolene, and much better for both when heated air was supplied. This fact in addition to that of liquid collecting in the cone, *B*, seems to indicate that a mist rather than vapor was the result of the air passage over the net. This fact is further proved by the working under kerosene without dropping of temperature such as would occur with evaporation. With a steady set of conditions this apparatus worked well as was noted, but like the injector devices, no great variation of the fire could be made. It was tried with petroleum, and the result showed a collection of residues in *A*, only the lighter volatile parts coming off. A little carbonization appeared at the nozzle.

26. All these devices depending on the vaporization of oil at some point have given great trouble from regulation when en-

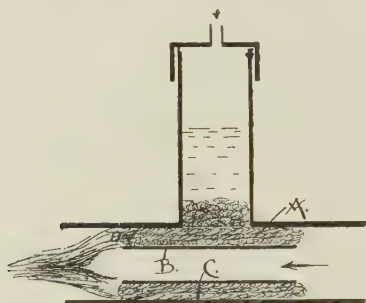


FIG. 9.

closed, and none was found satisfactory for variation of combustion pressure. It is probable that one could be designed, but it would necessarily be complicated. With the ones tried the temperature of the products could in no way be kept constant, and while most required large and variable excess of air, a few were found which could be operated by little; but the high temperature resulting invariably produced vapor decomposition. They required, moreover, special oils, but even this might be endured if a steady fire with always the same temperature delivery could be made to work under variable pressures; but these results could not be obtained.

27. The vaporization systems were now abandoned in view of these difficulties for the attractive simplicity of wick combinations which, while perhaps not offering the greatest perfection of combustion, yet would not go out when put under pressure. Fig. 9 was tried, with a bottom oil feed to the wick, and air sup-

plied above. It was found that the wick at the bottom of the chamber was not affected by pressure, and burned steadily when enclosed, but a steady discharge was necessary, for when the discharge was interrupted the flame was extinguished. It seems as if the products must be conducted away at once, and this is probably because, with the wick, the vapor generation will go on some time after the oxygen supply fails. It also seemed advisable to have the air current and flame tend towards the same opening and not oppose. Opposition produces a violent flame and irregular action which may cease entirely at any time.

28. To improve the means of renewing the atmosphere of this fire the burner of Fig. 10 was made. *A* is an asbestos mat supplied with oil from *B*. *C* is the air-supply pipe ending in the funnel *D*. If the oil be lighted at *A*, and time allowed for the whole to heat up, the burner can be enclosed and pressure applied

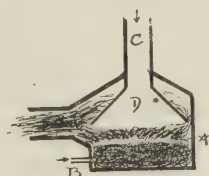


FIG. 10.

through the air-supply *C* without causing extinction. The pressure in the combustion-chamber has absolutely no effect on oil flame thus produced.

29. The products of combustion thus produced were piped to a small Shipman steam-engine to observe the effect of the impulse of engine admission on the action of the fire. Good results for any pressure were obtained with only one drawback. If the velocity of the air over the flame be too high, the flame will go out. With gas or oil previously vaporized a surplus as well as a deficiency of air will cause extinction; here, any surplus will have no effect, provided only that it move slowly enough. A most important result was here attained, viz., the flame could be kept going under working conditions.

For a more perfect and rapid combustion of oil by the wick method, it seemed desirable to keep the flame hot even beyond its visible part, and everywhere supplied with fresh air. This could be done either by drawing the flame out to a thin sheet, or by shooting across it warm air currents, as in the blow-pipe. Ac-

cordingly, the apparatus of Figs. 11 and 12 were constructed. In *A* is asbestos, on top of which the oil rests, and through which oil trickles to the part below enclosed between pipe, *C*, and surrounding pipe, *B*. The flame having been started at *I*, air was turned on through *C*; the flame was conical, with a well defined

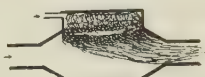


FIG. 11.

blue interior, and was blue even at the tip. This method of feed might be duplicated by dropping oil in varying quantities on the loosely-packed wick if a variable combustion is desired. This burner can be enclosed and put under pressure, and, moreover, unlike the last, is not sensitive to change of velocity of air through it.

30. While not all the conditions desired are here met, many

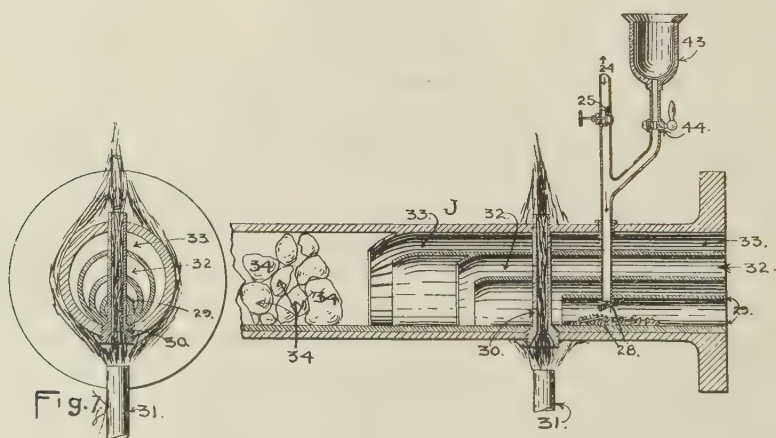


FIG. 12.

that are most important are fulfilled. The burner will work, enclosed, fairly steadily, and is not affected by pressure changes, but it always requires a large excess of air, and, therefore, delivers products whose temperature, though fairly constant, is yet not the maximum. Fig. 12 was made to be an advance on this type, by introducing a hot air blow-pipe effect. The flame here, instead of

having a blue center, has a deep yellow core forced by the air currents into blue at the edges. The center is the gas generator, which gas is completely burned at the edges by air from the heated lips of the tangent tubes. External heating to redness will cause internal ignition, and wicks placed in the path of the products seem to improve the action, both in completeness of combustion and as re-lighting after extinction. This burner could be used with satisfaction in every point, except that it used such large quantities of air and delivered products of comparatively low temperature.

31. The tendency of the preceding experiments is evident, always away from special vaporizers to arrangements with automatic regulation, the vaporization taking place in the firebox, in the presence of air if possible, and so preventing not only decomposition and carbonization, but also condensation. It seems rather odd that in general the means which worked best under the difficult conditions imposed were in general the simplest. For the use of all oils, including those of low and those of high boiling points, probably the following conditions would, if they could be fulfilled, produce a very good oil fire :

The oil to be introduced as liquid with the air and brought immediately, still with the air, to the hottest part of the fire, with means added to prevent the mixture of the vapor thus produced and its air from mixing with products of combustion of matter already burnt. At the time these conditions were formulated, it seemed impossible that any apparatus could be constructed which would permit of such action ; but in fact, such an apparatus was found, and worked so well as to entirely justify all the labor of classification and minute experimental observation, which made it possible to predict what conditions should produce a good method, even when the means seemed impossible to find.

32. This resultant method was not the outcome of this series of experiments alone, but rather of the combined oil and gas experiments, some of which have been previously reported. Just about the time the experiments above described were completed, and the probably necessary conditions for the good oil fire formulated, the explosive gas-fire described in the author's paper, No. 923, Vol. XXXIII., p. 292, was discovered.

By the use of the explosive mixture, a fire can be made in a closed chamber, requiring no atmosphere beyond the mixture fed, and such a fire will deliver hot gasses at a constant unvarying

temperature, no matter what the quantity burned; this temperature is the maximum possible, as no excess of air is heated; and, finally, this very excellent fire calls for no special apparatus, requiring simply an opening through which the feed must be made at a rate exceeding the rate of propagation, which opening is surrounded by a pile of broken rock. The function of this broken rock is to decrease the velocity of translation by increasing the area of cross-section of the advancing stream, and to increase the rate of propagation of inflammation by heating until these

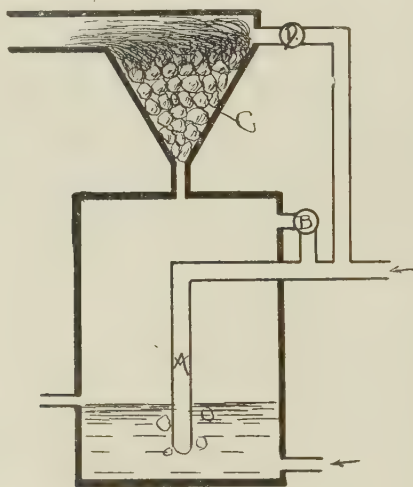


FIG. 13.

two rates become equal, allowing the combustion to localize within the fragments.

33. This suggested a revival of the experiments on oil along the lines laid down as follows: It was desired to vaporize the oil and produce with the vapor an explosive mixture which, in return, was to burn under pressure as desired in one of the explosive burners. The apparatus in Fig. 13 was constructed to do this; gasoline is held in a chamber kept at about 60 degrees Fahr., and bubbles through from *A*, the carburetted air was rendered explosive by the manipulation of the by-pass *B*, admitting pure air above the liquid. The resulting explosive mixture was burnt in the explosive-burner, *C*.

34. This arrangement fulfilled the requirements exactly so far as this particular fuel was concerned, giving a fire under pressure

not affected by any changes in pressure however sudden, and delivering at all feeds hot products of exactly the same temperature.

This burner was also piped to the steam-engine and a second by-pass, *D*, permitted, keeping the temperature of the air entering the engine under perfect control. Wide variations of speed and pressure had no effect, neither had the pulsation due to engine admission and cut-off.

35. Here, then, was a very encouraging result, but, unfortun-

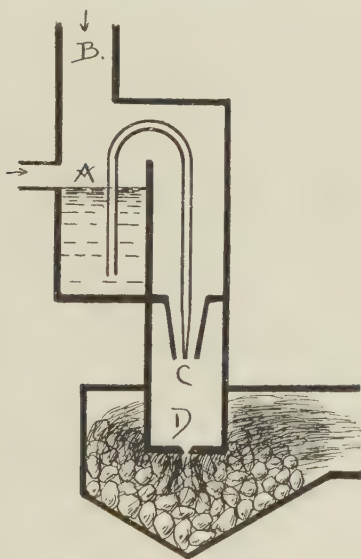


FIG. 14.

ately, only, the fuels easily vaporized, such as naphtha, gasolene, benzine, alcohol, etc., were available.

The next step was an attempt to utilize kerosene in a somewhat similar way. To this end the apparatus of Fig. 14 was set up. Here oil is fed to chamber *A* and kept at variable level; air is admitted at *B* and passing *C* throws a spray into *D* where it is vaporized by the heat of the fire; the end *D* being covered with the rock an explosive fire resulted, the correct proportion of air to vapor being maintained by varying the air supply and oil fuel.

Thus, while it worked under some circumstances and gave a very satisfactory fire, showed the same trouble that was experienced with sprays in the other series of experiments and was

abandoned for the device there found to be more satisfactory, *i. e.*, a surface boiling of the liquid. Fig. 15 shows the device constructed for this purpose.

36. Air was admitted at *A*, and with it, at first, some gas, making an explosive fire at *B*. The oil was fed from below to the cone under the plate, *C*, heated from above. Varying level

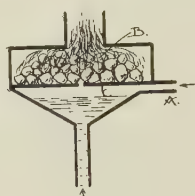


FIG. 15.

exposed more or less surface to be heated and varied the distance from the fire plate. The regulation in practice was not as good as one might expect from the device. The vaporization by approach of the oil to the hot parts suggested the next step which is so obvious that it seems as if it should have been tried first. This was to simply feed the oil and air through the same pipe to a pile of rock where the explosive fire is maintained, with the expectation that the hot pipe will do the vaporizing. The oil is fed through cock *A*, Fig. 16, and the air through *B*, both reach the fire through the same pipe *C*, and burn explosively in the

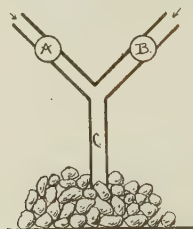


FIG. 16.

mass of rock. This was eminently satisfactory, and showed an action which was very fine, if unexpected.

37. When the feed is slow the pipe *C* becomes hot and then does undoubtedly act as a vaporizer, but when the feeds are increased the fire is forced away from the nozzle, as in the case of gases, and the pipe *C* remains almost cold, no matter how hot the fire in the rocks, but the perfection of the action is maintained

and it is found that not the pipe, *C*, but the hot rocks themselves act as the vaporizer. The air and oil impinge together on the hot mass, spreading out in constant velocity surfaces; the combustion takes place on that surface where the velocity is equal to the rate of propagation and in the passage the oil automatically vaporizes by contact with the same rocks which make the explosive fire possible, and all this happens without diffusion with the products of previous combustion. Thus the function of the rocks becomes complicated; first, starting with gas the explosive fire is made possible by their presence, and the result is the heating of the entire mass from top to bottom, the mass thus heated is a perfect vaporizer for the oil, which, fed with its air makes an explosive mixture and maintains the temperature of the rocks, the whole interrelated series of actions and reactions producing what I have named the "Explosive Oil Fire."

38. Were the proportions not explosive the interior of the mass would chill and the vaporization would stop. It is a very striking experiment to withdraw the nozzle from the intensely glowing mass of rock, of a properly working fire, and note the oil drip, drop by drop, giving off each time a dull red flash and a cloud of smoke, while the whole rock mass cools down; a re-insertion of the nozzle causes at once a resumption of the intense rapid high temperature combustion. And, secondly, by a simple change of proportion observe an instant cessation of the action, producing first smoke and then total extinction.

This method of burning the oil is perfectly adapted to the purpose for which it is designed, *i.e.*, the combustion of any oil in a closed pressure-chamber, as already described, and the action leaves nothing to be desired.

39. Naturally the next question would be to determine the action with residue oils of petroleum. It need only be remarked here that with every oil tried the action was the same; and three fires side by side, burning respectively kerosene, cylinder oil, and linseed oil, showed no difference in action. The so-called residue oils leaves no residue this way. The experiment of feeding the several oils successively through the same fire without interruption resulted in no apparent change of action. We can now see how the action of the brick that Kermide placed on his grates improved the action, which would have been still further improved if the spray had been prevented from diffusing with products before reaching the brick. Moreover, by this simple change

the spraying process would be rendered unnecessary. Also in the case of the saturated mat referred to in the earlier part of the paper, it will now be readily seen how the feeding of both oil and air through the same opening, instead of as designed, would have completely changed the action.

Originally firebrick fragments were used, but the fire was intensely hot, and fused such fragments together, even fluxing them and causing a flow. Later other rocks were tried, and magnesite was found not to fuse; dolomite, also infusible, crumbles slightly.

It was noted in the experiments on gas that considerable gas might be added to a mixture in excess of that required for chemical proportions without injuring the explosive properties of the resulting mixture, which fact was of value in producing surface flames above the explosive fires; of course, there will be a point where the explosive property will be lost and extinction ensue.

40. When an excess of oil was tried the explosive fire between the rock fragments, which act as the automatic non-diffusing vaporizer, was maintained by the lower and explosive part of the fire, while the excess of oil passed on to be burned above. It was found possible to vary the oil 100 per cent. without stopping the explosive action below, the effect being merely a variation in the length of the surface flame. This variation at will in the character of the surface flame is of no importance in the problem which was originally set, *i.e.*, the production of a fire for an internal combustion engine working by the increase of volume at constant pressure. It is, however, of the utmost importance in metallurgical and steam-boiler furnaces, and a few experiments other than those originally set were made on these applications.

41. Fig. 17 shows a series of burners which were used experimentally with success on both open and closed fires, showing the great simplicity that here meets with success. The one at the lower left-hand corner, shows an air chamber of 2-inch pipe through which the oil pipe is led, the air and oil passing downward at an incline of about 30 degrees to the rock bed. Proportions of mixture are maintained by external valves; the outlets may in this type be easily duplicated.

The one passing the stool shows a down bending quarter pipe fed with air and oil, and provided with a starting gas cock. About

one square foot of rock several inches thick can be kept in a glow with this.

42. To show that the theory of the formation of combustion surfaces holds with this oil fire as with the gas, a pile of broken brick was arranged to run about fifteen minutes, covered with clay; the result is shown on the top of the chair, a round cavity was fused out and a center lump left, showing what would be expected with

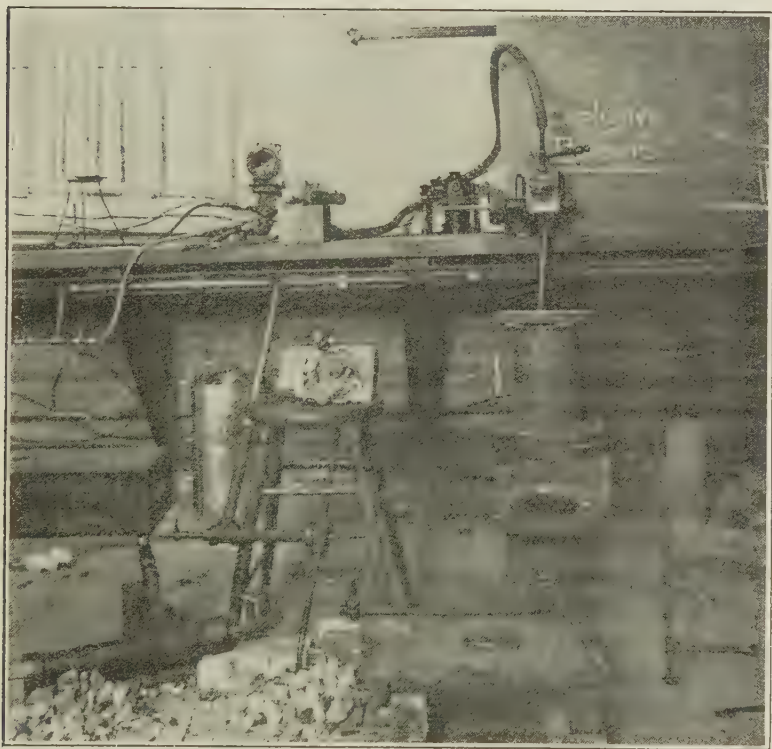


FIG. 17.

this down bending nozzle from the theory, viz., an annulus form of combustion surface.

The plate on the floor at the right is tapped to receive from below the nozzle lying on the top containing a center vertical oil feed surrounded by the air feed. This does not work well on a flat plate as some oil collects in a circle on the plate, where it meets little air, and is moreover chilled by the plate, a conical brick bottom works better.

Two 6-inch nipples arranged for closed fire-pots are shown, the lower one provided with oil, air, and gas inlets delivering to a pipe which merely enters the wall of the fire chamber. This works very well; after heating by a properly proportioned mixture, the whole becomes dazingly hot, and a blue to orange surface

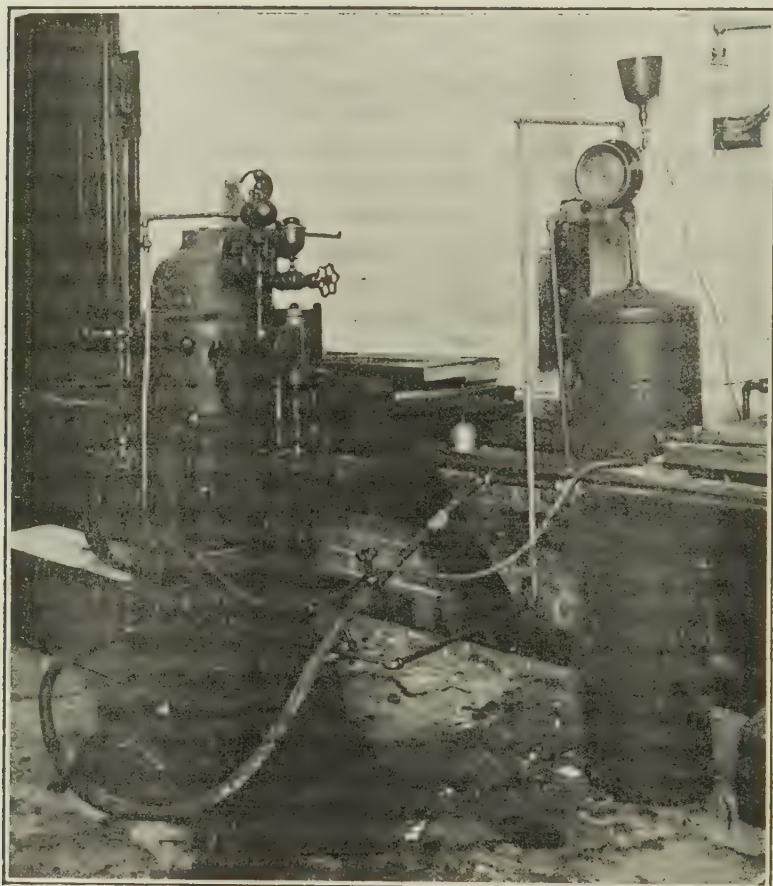


FIG. 18.

flame several feet high can be obtained without disturbing the action of the lower fire. While the whole firepot is white hot and would melt in time, the horizontal feed pipe is always cool enough to be borne by the hand. The very high temperatures that can be produced may be easily estimated when it is stated that this burner can consume a gallon of crude petroleum in about ten minutes, and in so doing uses no excess of air.

43. The upper nipple shown is connected for use with gas, and is provided with a 1-inch clay lining. It is fed from below with a mixture of air and gas from the motor-driven 6-inch positive blower. This was used for melting crucibles of tin, aluminum, lead, copper, etc., in the calibration of a Le Chatelier pyrometer for some experimental determinations of the passage of heat through metal from a hot gas to a cold.

44. In the center of the shelf is shown a $2\frac{1}{2}$ -inch cross bottom fed by air and gas direct from the mains and used for heating soldering irons. Kerosene has also been used in this apparatus for the same purpose.

The first application of this method of combustion to a steam boiler is illustrated in Fig. 18. The oil tank in the rear has a delivery pipe starting at the bottom of the tank, and air from the



FIG. 19.

main is piped to the oil surface and to the burner through the hose; a slight throttling at the boiler will put enough pressure in the oil to lift it to the burner, where it passes the valve seen in the right front. A half-inch pipe leads to the center of the firebox and then turns down by an elbow; the grate is covered with clay and the firepot filled with broken rock. A gas connection is shown for initial heating and a steam pipe passes from the dome vertically downward in front. It was for observing the action of steam in the fire that this particular apparatus was set up. It was hoped that by the decomposition of the steam in the fire the excessively high temperatures would be avoided and the use of special rock of high fusing point rendered unnecessary. If the fire be started and brought to a steady glow and steam be then admitted, there will at once appear an almost invisible surface flame showing the action desired, a decomposition of steam in the hottest parts and a recombination, more or less complete, beyond at the surface. The steam is thus a sort of heat distributor, and

in this way it was found feasible to use common fire-brick; an occasional sticking together is easily broken up at the end of a run by a bar, and everything made as good as new.

45. It was also thought worth while to try what could be done in producing reverberatory action, similar to that of coal fires. To this end, this apparatus, Fig. 19, was made of brick and clay. With a 5-inch fire at *A*, and an air inlet at *B* and *C*, a good hot-colorless flame 2 feet long could be produced, heating the chamber *D* to an even glow with an atmosphere reducing or oxydizing as desired.

It should be noted that all of the explosive burners described will work under any air pressure whatever, a variation merely altering the distance of the combustion surface from the outlet, but for burning a given amount of oil a larger air pipe must be used with low pressure air feeds than with high.

46. In conclusion, it may be said of the method resulting from this experimental research that it seems to be in every way satisfactory for the purpose for which it was derived, and may be of use in other applications. It has no small openings for oil, no possibility of carbonizing; will burn any oil with air at any pressure, provided only that enough air be supplied, and is subject to an almost unlimited variation of form; it will deliver gases at a constant and maximum temperature, which may be lowered to anything desired by air dilution; is capable of burning more oil in less volume than any of the other forms tried, and this with the least possible amount of air. It must be stated as a drawback that without the use of steam it calls for the use of selected rock to prevent fusion.

PHYSICAL PROPERTIES OF EXPLOSIVE MIXTURES.

Power generation, involving as one of its phases the internal combustion method of heating a gas, demands a knowledge of the properties of explosive mixtures not only qualitatively, but quantitatively as well. For, internal combustion presupposes the fuel and requisite air in proper or otherwise known proportionate amount introduced into the closed system, and investigation has shown that to obtain the best results in heated products of combustion these two elements—the fuel and air—should be mixed before combustion, producing thereby an explosive mixture. Moreover, it has developed that, no matter whether the fuel be liquid or gas, the explosive combustion of the resulting explosive mixture is not only the best from the point of view of physics, but also from that of simplicity and practicability, that is, it is not only the best way, but it is the simplest and easiest to carry out.

Researches by very eminent scientists on this subject have shown: (1) That explosive mixtures have properties not possessed by other mixtures, (2) they have pretty well developed the nature of these special qualitative properties and (3) they have measured the extent and intensity of many of these physical reactions of these mixture. But in spite of the information developed by these men the fact remains that to-day, when we are so extensively using explosive mixtures in our exploding gas-engines and contemplating their utilization in other ways, there does not exist data sufficient for the calculations of many of the quantities needed, nor is there obtainable apparatus sufficiently reliable and practicable to enable designing engineers to obtain the data needed for their work. It has been the aim, then, of this part of the work not only to work out if possible a properly simple and accurate means for obtaining such data, but also to use the apparatus in the making of such observations as time might permit. Before entering into the work forming the subject matter of this chapter it seems advisable to first look at the work of the scientists referred to and to note their results.

Bunsen, in the course of his work on gas analysis considered:

(1) The heat of combustion of a gas; (2) the temperature of combustion; (3) "the explosive force of gases"; (4) temperature of ignition of gases, and (5) limit of inflammability of mixtures, as influenced by dilution. All of these were undertaken chiefly in reference to one phase of his system of analysis, *i. e.*, the determination of combustible gases in a heterogeneous mixture under analysis.

The heat of combustion he calculates from that of elements determined by analysis, using the elemental values of Favre and Silbermann. The temperature of combustion and "explosive force," or pressure after constant volume combustion, are calculated from this last by assumption of a constant value of C_v . Under temperature of ignition he simply noted that a gas which, by reason of dilution, became unflammable regains its combustibility if prevented from expanding freely during ignition or when its temperature is raised. Under limit of inflammability it was observed that inflammable mixtures might be rendered unflammable by dilution, and that the point of difference is sharply marked. His table is:

1 Volume of detonating gas with	{ 2.82 CO ₂ is inflammable.
	{ 2.89 CO ₂ " non inflammable.
1 Volume of detonating gas with	{ 3.37 H is inflammable.
	{ 3.93 " " non inflammable.
1 Volume of detonating gas with	{ 9.35 O is inflammable.
	{ 10.68 " " non inflammable.

He also determined some rates of propagation of explosive mixtures with the pressure tank and orifice method; results later shown to be quite erroneous.

Following him, whatever was done by other individual investigation was overshadowed by the work of Berthelot and Vielle and Mallard and Le Chatelier. The only extensive work undertaken with the sole object of studying the properties of explosive mixtures was that of Mallard and Le Chatelier, published in various papers, and collected and republished in the collected works of the "Commission de Grison," 1883, with the title "Recherches Expérimentales et Théorétiques sur la Combustion des Mélanges Gazeux Explosifs par MM. Mallard et Le Chatelier Ingénieurs au Corps des Mines."

They say in the introduction: "We have not limited our work to mixtures formed by air and fire damp; we have extended it to

the principal combustible mixtures. We believed that we should thus be able to profit by apparatus often costly, set up by us, and by the experience gained in its manipulation to furnish to science some new facts on questions still but little known" (1883).

This work and notes on that of previous investigations, which is reported in some three hundred pages and several plates of cuts, was divided into three parts:

1. Conditions necessary for starting active combustion and the temperature of inflammation.

2. The rate with which inflammation, once started, will propagate itself through the gaseous mass, and in general the circumstances characterizing that propagation.

3. The pressure produced in a closed vessel after the combustion of the gaseous mixture enclosed in it, from which can be deduced: (a) Law of cooling of hot gases in cool walls (b) temperature produced by combustion; (c) nature of variation of specific heat at high temperature.

The subject of temperature of inflammation is treated as follows:

Historical.—The work of Davy, who observed that, at times, when a metal bar while hot might not inflame mixture a flame will. He arranged some gases in order of inflammability: Marsh gas, ethylene, carbonic oxide, hydrogen and phosphoretted hydrogen. To the last, PhH_3 , he gave 116°C .

Davy also noted that slow combustion unaccompanied by heat and light always took place in mixtures.

After him, Bunsen, who worked on questionable theoretic grounds, gave these figures:

1 Volume (H + O) + 2.85	Volumes of CO_2 . . .	1790°C .
" + 3.65	Volumes " H	2116°C .
" + 10	Volumes " O	857°C .

Nothing more was found, probably due to difficulties.

Method of Experimenting.—After considering several methods all are rejected as inaccurate or impracticable except the one adopted. Mixture is admitted rapidly into a chamber—previously heated to a known temperature—which may be empty or filled. Both methods were used. It is then observed whether the gas ignites or not, and two limiting temperatures can be determined

between which the temperature of ignition must lie. It was found very slow work and difficult to avoid both accidental and systematic errors; however, results were tabulated for mixtures of H and air in all proportions and diluted with CO_2 and O. Similarly for CO and C_2H_4 , fire damp. The limits in the three cases are for all mixtures:

H,	517–595°, mixed with air, O and CO_2 .
CO,	630–725°, mixed with air, O and CO_2 .
C_2H_4 ,	640–760°, mixed with air and O.

Experiments on slow combustion show a discontinuity between it and that accompanied by light and heat changes.

The whole is summarized as follows:

The temperature of inflammation can be fixed at

555°	for explosive mixtures of H and O.
655°	“ “ “ “ CO and O.
656°	“ “ “ “ C_2H_4 and O.

The addition to explosive gas of even a considerable volume of inert gas modifies little or not at all the temperature of inflammation.

However, with mixtures of CO and O the addition of notable quantity of CO_2 seems to elevate that temperature to a sensible degree. One volume of CO_2 added to explosive mixtures $\text{CO} + \text{O}$ raises the temperature from 655° to 700°.

For mixtures in which H and O are the elements the combustion takes place as soon as the temperature of inflammation is reached. It is entirely otherwise for marsh gas, which we may liken to fire damp. The mixtures formed by this gas with air or oxygen do not burn except after having been brought to and kept ten seconds perhaps at a temperature equal to or superior to that of inflammation. The retard of inflammation increases with difference of temperature of gas and that of inflammation and with the increase of inert gas. This latter reason explains why, according to Davy, a bar of red-hot iron, though above 650°, will not ignite a mixture of fire damp. By opposing circulation one may easily provoke inflammation because when it circulates freely the gas does not remain long enough exposed to the temperature of inflammation.

RATE OF PROPAGATION.

Davy is recognized as the first to study the question. Without measuring exactly he knew it took less than one second for the flame to travel through the best mixture of air and fire damp one foot long, and also came to the conclusion that small diameter tubes and metal gauzes will prevent passage of flame of the majority of mixtures.

Bunsen is noted as having found, by his orifice and tank method, the figure of 35 m. per second for H and O.

MM. Schloesing and De Mondésir did some unpublished work for gas-engines, observing in glass tubes the progress of the flame. They used mixture of CO where r is so slow as to be easily followed by the eye. They noted that for these slow mixtures an agitator such as that of the jet of gas into a quiet mass makes r very great and that combustion itself causes many agitations, so that the values observed may vary widely and be always different from normal. The agitations are due to: (1) Difference of density between burnt and unburnt gases; (2) dilatation of burning part; (3) vibratory actions of several kinds due to compressibility of gas when subject to impulses.

M. Fonesca experimented with mixtures of O and various gases that burn with it. A stream of mixtures is given a high velocity till the flame cap rests some distance from the orifice; the velocity is then reduced till contact occurs and some figures deduced.

H + O	35 m. per sec.
CO + O	1.40 "
C ₂ H ₄ + 8O	2.10 "
PhH ₃ + 8O	9.20 "

M. Gouy tried to deduce r from the angle of the luminous cone in Bunsen flames. Berthelot and Vieille worked on the subject and found the rate of propagation abnormal and extremely high, for certain cases moving several thousand meters per second—very superior to sound. They called this mode of propagation the explosive wave and recognized that the wave itself must travel with the velocity of sound.

With this experience to guide them Mallard and Le Chatelier began work. Though the method of orifice was recognized as introducing many errors and as more or less dangerous, it was

employed for mixture where r did not exceed one meter per second. The orifice was 0.01 m. in diameter. The second method was that of a tube closed at one end and open at the other with ignition at the open end.

Time was measured by automatic machines—electric, pneumatic and photographic. The electric depended on a passage of a spark through the flame when gap was too large for passage through cold gas. The pneumatic depends on the explosion of gas in chambers connected with the tube and ignited by the passage of the flame. The photographic consisted of a moving plate receiving the action of the flame in a glass tube giving a curve whose abscissæ are distances in tube and ordinates time. All these methods called for a delicate and expensive apparatus and the results obtained are not likely to be soon duplicated.

It was found that various influences acted to change the rate of propagation and figures are given for each.

1. The material composing the tube

$$\begin{array}{l} \text{[CO + O] in .01 m. tubes} \\ \left\{ \begin{array}{l} \text{For glass} \quad . \quad . \quad . \quad 2.20 \\ \text{For lead} \quad . \quad . \quad . \quad 2.35 \end{array} \right\} \text{ m. per second.} \end{array}$$

2. Diameter of tubes containing the mixture. One limit is

$$r = 3.00 \text{ m. per sec.}$$

$$D = .003 \text{ m.}$$

3. The temperature of the mixture H and air with 30 per cent. H.

$$15^\circ - 3.28 \text{ m. per sec.}$$

$$100^\circ - 4.35 \text{ m. per sec.}$$

4. Nature and proportions of mixtures in tubes .01 m. in diameter:

H per 100.	Velocity.	C ₂ H ₄ per 100.	Velocity.	11.65 (C ₂ H ₄) + x.	Velocity.
6	.00	5.6	0.00	.5 N	.42
10	.60	6.0	.03	1.0 N	.30
20	1.95	10.0	.42	1.4 N	.19
30	3.30	12.0	.61	.5 CO ₂	.31
40	4.37	14.0	.36	1.00 CO ₂	7.16
50	3.45	16.0	.10		
60	2.30	16.2	0.00		
70	1.10				
80	0.00				

Illuminating gas and air :

Gas per 100.	Velocity.
10	.44 — .48
12	.68 — .84
15.0	1.02 — 1.05
17.5	1.16 — 1.21
20.0	.88 — 0.98

H and O give rates from 40 — 480 m. All the above concerns only the uniform movement, but this is always followed by the vibratory movement and later by the explosive wave if the mass be large enough and sufficiently extended.

A summary of the work on propagation of inflammation brings out the following facts :

There are two modes of propagation: (1) Normal, that by conductivity, and (2) that which takes place by the transmission of a pressure sufficiently high in the propagation by explosive wave. These correspond to deflagration and explosion of dynamite, etc. Each has a fixed velocity for a given mixture at a given pressure.

R due to normal propagation never exceeds 20 m. per sec.

For H and air the maximum is 4.30 m. per sec. for a 40 per cent. H, *i. e.*, an excess (30 per cent.).

For C_2H_4 and air the maximum is 0.62 m. per sec. for a 12.2 per cent., *i. e.*, an excess (9.4 per cent.).

For illuminating gas and air the maximum is 1.25 m. per sec. for a 17.0 per cent., *i. e.*, an excess (15 per cent.).

For CO and O and air the maximum is 2.00 m. per sec. always.

R increases with I and when tube is large is independent of diameter, but a tube small enough may cause extinction.

Agitation increases R . Combustion in tube with slow R sets up oscillation which may cause extinction.

When for any reason of vibration or explosion of burnt gas the pressure transmitted to a layer next is equal to that which would elevate it to the temperature of inflammation, the combustion propagates with the same velocity as the compressive wave resulting in the explosive wave.

TEMPERATURE OF COMBUSTION.

Dulong, Favre and Silberman, Thomsen and Berthelot all worked on Q , from which the temperature of combustion was to

be calculated with a known value of C_v the specific heat. But Saint Claire Deville showed that dissociation could take place. He tried dropping hot metal from the flame to water. Crova and Rosetti used optical methods on flames. Vieille used spherical bombs and noted displacement of piston to get maximum pressures.

Bunsen, among others, tried to compute the temperature of combustion from observed values of pressures resulting from explosion. Some of the pressure ratios determined by him follow :

Gas Added to 1 Volume of Explosive Mixture.	$\frac{p_2}{p_1}$ For Mixtures of H.
0	9.97
0	9.75
1.26N	7.49

Gas Added to 1 Volume of Explosive Mixture.	$\frac{p_2}{p_1}$ For Mixtures of CO.
0	10.78
0	10.19
.108O	9.05
.686CO	8.89
.855O	8.44
1.086O	7.86
1.256N	7.73
1.256N	7.35
1.71O	6.67
2.16O	5.83
3.16O	4.79

RÉSUMÉ OF TEMPERATURE OF COMBUSTION

Pressures developed are higher than the static due to the heat developed. Before communicating itself to the whole mass the increase of pressure concentrates itself on the layer in contact and the effect is greater the greater the rate of propagation.

Permanent gases cool according to

$$\frac{d\theta}{dt} = ae + be^2$$

when e is (temperature of gas)—(temperature of walls) ;

a is independent of pressure.

b is inversely proportional to density.

If the gas can condense the fall in pressure is given by

$$\frac{dw}{dt} = a(w - p_0).$$

w is the variable pressure.

p_0 is tension of the vapor at temperature of the walls.

When gas is partially condensable we have

$$\log \frac{1}{w - p_0} \frac{dw}{dt} = a(w - p_0) + b(w - p_0)^2,$$

where w is the variable pressure and p_0 is pressure of whole mixture at temperature of walls.

Dissociation.—Information from cooling curves. Dissociated CO_2 will recombine when mean temperature of gas reaches 1800° . Gases mixed with it have apparently no effect. No dissociation of H_2O noticed.

Temperature of Combustion at Constant Volume.—Calculated from pressures when dissociation = 0, *i. e.*, from pressure ratios.

Specific heats are found by the method noted to follow closely these formulæ

		C_v .
For CO_2	$C_v = 4.33 (T/10^2)^{.867}$	$0^\circ - 2000^\circ$
	$C_v = 4.74 \sqrt[3]{T/10^2}$	6.3 — 13.6
For H_2O	$C_v = 5.61 + 3.28 T/10^3$	5.6 — 12.2
Perfect gases	$C_v = 4.8 + .0006 T$	4.8 — 6.0

Temperatures of combustion at constant pressure are calculated by making change in the value for specific heat.

After these facts were obtained there has appeared periodically attempts at development of special points. Dugald Clerk obtained some pressures due to explosion, and more recently others, including the Massachusetts Institute of Technology, have given some values of pressure ratios for mixtures of fuel and air, but the results do not always agree and seldom cover working conditions.

After all the work is looked over and the labor and expense attached to the results realized it seems rather a pity that we have nothing of any immediate value for the designer of gas-engines or the user of explosive mixtures in other fields. For example, it

is absolutely impossible to calculate the maximum pressure that may result in a cylinder of a gas-engine, even when the composition of the gas is known, or secondly to determine the change in volume due to the combustion of a mixture at constant pressure. Of course a calculation can be made, but it will be far from that realized by actual trial and the reason can no doubt be found in the great complexity of the process involving many unknown influences.

As another illustration of the unavailable form of much of the present information and apparatus there may be cited the case of determining data for the mean effective pressure that must be counted on in designing an exploding gas-engine.

Assuming the compression and expansion lines as constant curves the mean effective pressure of an Otto cycle card will depend on the compression, *i. e.*, cylinder clearance, and on the length of the explosion line, *i. e.*, on kind of fuel and composition of mixture. From the clearance can be computed the amount of burnt or partly burnt gases that will be mixed with a fresh charge, and the resulting complex mixture will have a certain pressure range for its explosion and this, moreover, for that mixture must be constant. But with the information at hand this question fundamental to engineers designing gas-engines cannot be computed.

The questions set down for clarification are these:

1. Pressures resulting from constant volume combustion of a mixture of gas with air in all explosive proportions to determine (*a*) best mixtures and compare with chemical determinations of the same, and (*b*) the maximum pressure for each mixture with variation due to change of composition.
2. Volumes resulting from constant pressure combustion of mixtures.
3. Heat of combustion of these mixtures burnt at constant pressure.
4. Heat of combustion for constant volume combustion.
5. The effect of dilution by products of combustion on all of these quantities.

These questions called for the design of apparatus:

1. For measuring air, gas and neutral products of combustion.
2. For mixing, compressing and storing mixtures.

3. For producing products of combustion by methods available for determinations on an engineering scale.
4. A constant pressure combustion calorimeter for explosive mixtures.
5. Same for constant volume.
6. A chamber for determining pressures due to explosion.
7. A chamber for determining volume increase due to constant pressure combustion.

With the apparatus at hand there are few of the questions vital to engineers entering into the thermal properties of explosive gaseous mixtures that cannot be determined in a way *immediately* available, *i. e.*, without computation, for use in design. It must be noted that, while many of the actions and processes are complex and a pure scientist would be bound to analyze them and assign to each element a value, the engineer is more concerned with resultant effects than elemental ones, and is, moreover, saved the possibility of multiplied error in computing the resultant from the elemental if the resultant can be measured directly with sufficient known conditions to insure constancy and serve as a specification for the process.

SOME NEW WORK ON PROPERTIES OF EXPLOSIVE MIXTURES.

THE APPARATUS.

All work in the experimental study of heat is, as is well known, very difficult, calling for most careful observations with apparatus sometimes impossible to construct with sufficient accuracy, and always expensive even to a slight degree of accuracy. The study of the characteristics of explosive mixtures is no exception to the rule, partaking of the general difficulty of all heat work, that of isolation of phenomena of observation, and the prevention of the manifestation of more than one at a time. In most cases each experimenter has designed and constructed apparatus of his own, and in no case, it seems, has any one used instruments first employed by a predecessor. It is probably due to this that the results of different observers do not always agree. In none of the researches does there seem to have been adopted a sufficiently direct and simple means for obtaining ALL the results; having an instrument to measure one constant the observer has rested content with COMPUTING others equally important, which were mathematically related, though these computed values might also have been observed directly. As actual trial has shown that the results obtained by this process of computation of constants from observed values of some other related constant is not always reliable, introducing, as it generally does, a multiplied error *if not involving unproved assumptions of interrelation*, it seemed desirable, (1), that each constant needed be measured directly under well-defined conditions, and (2), that the apparatus for the measurements be made simple enough for duplication by others whose observations could act as a check on the results. In this reduction to simplicity it is essential that the apparatus be so constructed as to permit of measurements on the mixtures under as nearly the same conditions as found in engineering practice as was possible. Such a set of apparatus once set up in a laboratory can be used to rapidly determine what is wanted from time to time as occasion might demand, and the results could easily be checked up by other ob-

servers on the same or duplicated apparatus. The actual construction should involve little or no machine work, and the instruments of observation must be those in common use in the general engineering laboratory.

The results of the work on continuous explosive combustion of gaseous mixtures furnished a ready means for obtaining burnt gases for dilution of mixtures of otherwise known composition of air and gas. The action and appearance of the fire itself furnishes a very good and simple means for determining the propor-

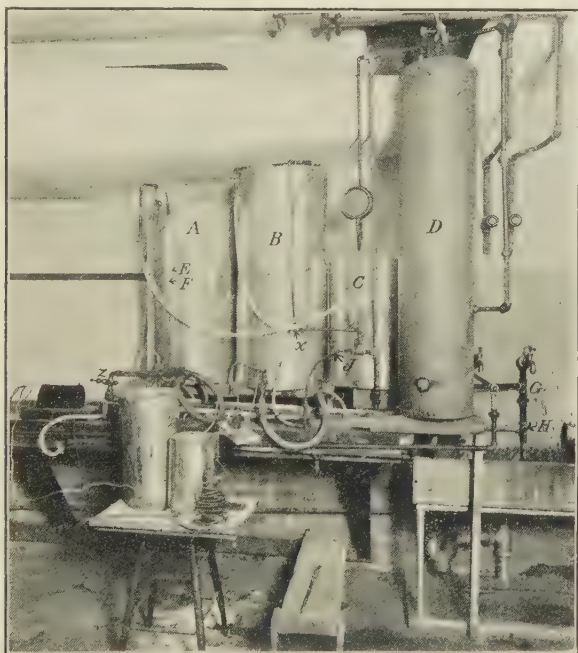


FIG. 1.

tions being fed, and hence the character of the products of combustion, whether oxidizing reducing or neutral, within limits pretty close; much closer than we can attain to constancy in gas-engine work.

For a constant pressure combustion-calorimeter this same method also filled requirements. For, as has been shown, a mixture of air, gas and neutral in any proportions that can be exploded may be burnt in a closed chamber and under pressure, so there

only remains as a necessity, the provision for measuring gas burnt and heat developed by submerging the whole apparatus in water. The details of this calorimeter as adopted and used will be noted later.

Gas measuring, mixing and storing was accomplished by water displacement in thin metal tanks; the quantities read by the levels in a water glass before and after more water had displaced the gas required. All connections and communications between the parts were made of combinations of pipe fittings and rubber hose. Four tanks were provided. (Fig. 1, *A, B, C, D*), each fitted with water inlet and discharge communicating with water main and sewer, and each fitted also with gas inlet communicating with the appropriate source and discharges connecting *A, B*, and *C* with *D* and that of *D* with the apparatus in which mixtures were to be used. The first three tanks then were measuring tanks, *A*, for illuminating gas, *B*, for air, and *C* for products of combustion prepared previously, while the fourth, *D*, is the mixing and storage tank and is

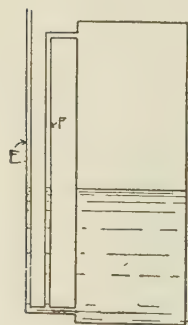


FIG. 2.

the only one in which the pressure was allowed to exceed atmosphere. Two glass tubes, *E* and *F*, were provided to each of the measuring tanks and extending the entire length. One of these tubes, *F*, was connected by both ends to the interior, while the other, *E*, was connected at the bottom, only the top being open to atmosphere, Fig. 2. The doubly connected tube then serves as an ordinary water column showing the water level, while the other gives an indication of the interior pressure. When the level of the open column is equal to that in the closed column the pressure

on the interior is atmosphere; if the level of the open column rises above that of the closed column then the interior pressure exceeds atmosphere by an amount exactly equal to that due to difference of level.

To obtain a mixture of any composition the method of operation was as follows: All tanks were filled with water till they overflowed; then, one at a time, the water was allowed to flow out of the measuring tanks *A, B, C* and illuminating gas, air and products of combustion, flowing in each to its own tank, filled the

space left by the receding water. Manipulation of the gas discharge and water inlet valves, guided by the relative position of the water levels in the two columns, enabled the operator to keep the pressure on the inside equal to atmosphere, and relieving the walls of any stress as well as preventing compression or rarefaction of the gas. When each tank is filled with its respective gas the amounts of each desired in the mixture are laid off on the water glass above the present water level and, as these have all the same diameter, the height on the columns will be a measure of the quantity of the gas. Water is then turned out of the storage tank *D* and a partial vacuum created therein. With one hand on the gas cock *x* and the other on the water cock *y*, the amount of gas wanted is caused to flow from tank *A*, where it was measured, to the tank *D*, where it is wanted, the transfer being made without allowing any change of pressure in tank *A* by simply regulating the relative openings of the valves *x* and *y* and watching the two water columns. After the gas wanted is transferred air is similarly measured and transferred, and later, if desired, products of combustion, until finally the tank *D* contains all the constituents of the desired mixture. All openings to *D* are then closed and water from the main *G* admitted through the three-way cock *H*, until the full pressure (in this case 60 lbs. gauge) has compressed the mixture. This method of feeding and afterward compressing, results in a perfectly uniform mixture, as was proved by comparing effects derived from burning parts first drawn off with the last that remained.

We have now in tank *D* a perfectly uniform mixture of known composition, compressed to 60 lbs. and available for whatever tests or experiments we may desire to make, by the simple opening of the valve *J*. All this measuring, transferring and compressing the constituents of the mixture takes about five minutes from the beginning up to the time the mixture is ready for use. When the operator is working with one mixture this time may be lessened if an assistant is at hand to recharge measuring tanks and get the quantities desired measured off for the next mixture desired.

NEUTRAL GAS GENERATOR.

To generate neutral products of combustion a positive blower *A*, Fig. 3, driven by a Crocker-Wheeler motor *B*, fed an ex-

plosive mixture of air and gas to the two-inch tee *C*. The proportions were obtained as desired by air-cock *D* and gas-cock *E* on the blower suction and the mixture thus obtained burned within the mass of broken magnesite in the tee *C*. Ignition was effected through the opening *G*, and when the proportions were found correct by observing the fire this opening was closed, thus sending the products of combustion over through pipe *J* to bell *K* under

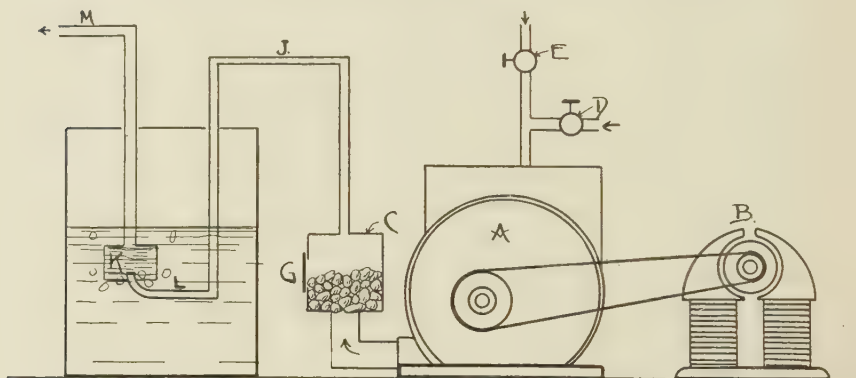


FIG. 3.

water in tank *L*. When wanted the products of combustion could be drawn off from *K* through pipe *M* under a constant pressure, being that due to the height of water above the bell. More gases than were wanted were continuously generated, the surplus always bubbling off from bell *K*; this was to insure getting fresh gases delivered at constant and small head. The water around the pipes served to condense and catch any steam in the products.

CONSTANT PRESSURE COMBUSTION GAS CALORIMETER.

The constant pressure calorimeter consisted of a one-half-inch tee *F*, Fig. 4, nearly full of broken magnesite and fitted with a jump-spark plug *G* operated by a vibratory primary circuit breaker induction coil. Explosive mixtures of the previously determined composition and of known amount were fed to the bottom of the combustion chamber *F* through a one-eighth-inch copper tube *E*. As it was necessary to discharge the whole of the measured quantity of mixture from the pressure tank *D*, and necessary secondly that no water should follow, the trap *B* was introduced.

It is simply a glass bottle with two glass tubes, one a feed and the other discharge, fitted to a perforated cork. The discharge tube was connected with the calorimeter by the light rubber tube *D*. Hot gases from *F* were discharged through *H* to the bottom of the water tank and thence passed up through square coil of one-

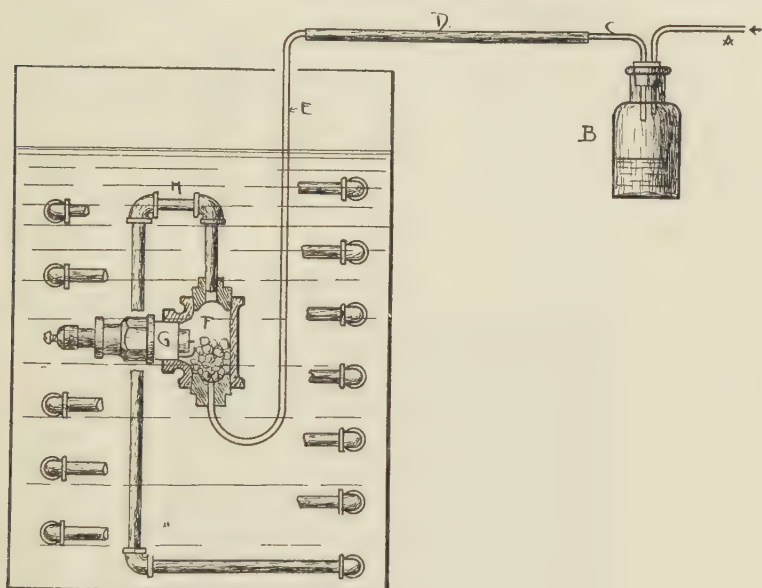


FIG. 4.

eighth-inch iron pipe. The end of this coil was supplied with rubber and glass tubes, so that the products of combustion could be directed to the atmosphere or to any point of the water to act as a stirrer when so desired.

CONSTANT VOLUME COMBUSTION PRESSURE RATIO CHAMBER.

The explosion chamber for determining the pressures due to constant volume combustion consisted primarily of tee *A*, Fig. 5, nipples *B* and caps *C*, *C'*. A Crosby gas-engine indicator was attached to the tee as shown. The spark plug *D* was carried in one branch of the three-way cock on top and on other branch was connected to mixture feed and water overflow openings *E*. The apparatus was first filled with water through valve *F* until it overflowed through valve *G*, the mixture feeding valve *H* and

water discharge valve *J* being closed; the spark points meantime being protected from the water by the three-way cock. When entirely full of water valves *G* and *F* are closed and *H* and *J* opened, the mixture from tank *D* thus expelling the water; the three-way cock is then thrown to permit contact of points with

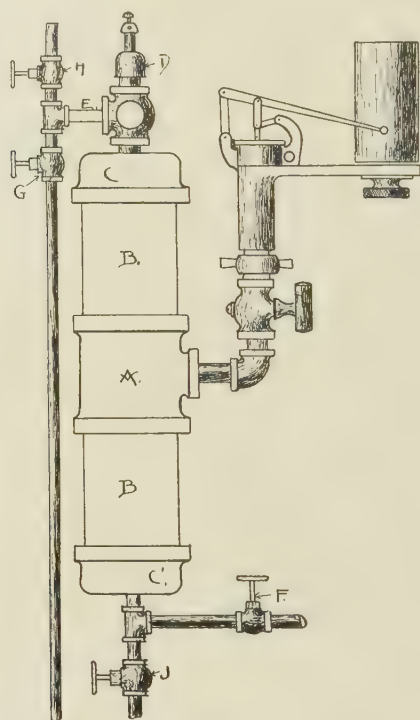


FIG. 5.

mixture, which is allowed to blow through freely to fill chamber at atmospheric pressure. Then all openings are closed and the mixture fired, the pressure rise being shown by the length of line drawn on the indicator drum to the proper scale. To expel the burnt gases water is admitted as before and the new charge subsequently used to drive the water out.

CONSTANT PRESSURE COMBUSTION VOLUME RATIO APPARATUS.

The quantity of gas that could be stored in the tank *D* is so small and the time to attain maximum effect in a heating chamber

so long, that with these tanks the combustion chamber could not become heated sufficiently to make a measurement of maximum volume increase. The apparatus of Fig. 6 was constructed with this end in view. It depends for its action on the principles of gas flow through an orifice. The rate of flow of a gas through an orifice is proportional to the form of orifice and to the pressure drop through the orifice. Now if the gas be caused to pass through a hole in a plate before combustion, and later, after combustion, pass through a similar hole in a similar plate, the constant due to the form of orifice would be eliminated in comparing velocities through the two holes. Secondly, when the fall in pressure through each hole is the same the velocity of flow through each plate will be equal, and the volume passing will be propor-

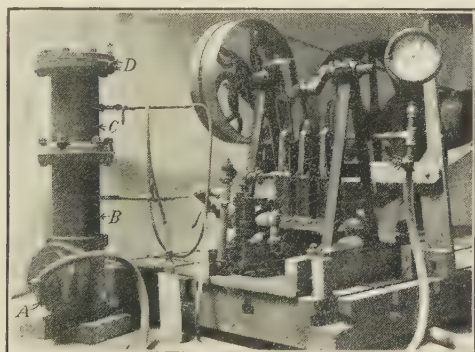


FIG. 6.

tional to the area of the orifice only if the pressures used be small enough to make correction for compression vanishingly small. Gas and air are mixed in any proportion desired at the compressor intake and delivered, mixed, to the chamber *AB*, from which the mixture will pass to the upper chamber *C* through a hole in the plate secured between the flanges. In chamber *C* there is placed a cone of brick to keep lower plate cool, and in the cone broken rock to permit of the combustion of the explosive mixture. The top plate between the flange *D* is provided with asbestos sheets to keep the hot gases from chilling just before issuing.

At times both the brick cone for the lower and the asbestos sheet protection for the upper plates were removed for the taking of

observations, while at another time a one-inch lining of fire clay was supplied to prevent radiation. Mercury manometers to both chambers indicate the interior pressures, and hence the drop in pressure through each plate.

RESULTS OBTAINED WITH APPARATUS. FLAMES IN ATMOSPHERE OF DIFFERENT AIR-GAS MIXTURES.

Before proceeding to the effects of the combustion of different mixtures it is necessary to first determine the limits of combustibility, and in so doing opportunity was afforded to observe the characteristics of the flames of different mixtures.

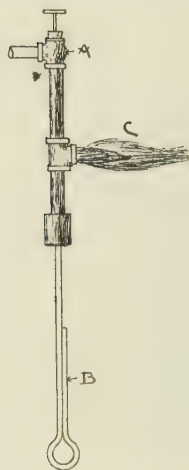


FIG. 7.

The mixture from the compression storage tank *D* was led to the apparatus, Fig. 7. This consisted of a one-quarter-inch tee with a valve *A* and manometer *B*, the flame locating at *C*. Mixtures were ignited and the flow regulated to determine the maximum length and character of the flame and the pressure at which the flame would blow off.

Appearance and blow-off pressures of mixtures of air and gas burnt at opening of one-quarter-inch pipe in atmosphere of air:

$$\text{Mixture, } \left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 1 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$$

Blow-off pressure, 1.25" H₂O

With a length of one-half inch a clear blue flame results; an increase to three inches in length develops a green core and faint spots of yellow appear. A still further increase of the pressure causes the core to become less distinct and the end of the flame becomes wavy and oscillatory. A roaring noise develops also. Just before blow-off the flame becomes violet and green near nozzle; the end is quite wide and spreading. Blow-off occurred at 1.25 inches of water pressure with a length of fourteen inches after some trembling at the nozzle.

$$\text{Mixture, } \left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 2 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\} \text{ Blow-off pressure, .8" H}_2\text{O}.$$

Flame all blue and remained blue as length was increased with

very much less spreading at the ends. There appeared no core. A length of twelve inches was the maximum at a water pressure of eight-tenths of an inch.

Mixture, $\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 3 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$ Blow-off pressure, .5" H₂O.

First appearance of the flame cap, which was of yellowish color surrounded by light blue and could be extended to a maximum length of six inches with a blow-off pressure of one-half inch water. The cap, however, instead of being smooth, had serrated edges.

Mixture, $\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 4 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$ Blow-off pressure, .3" H₂O.

The flame cap is now more distinct, with a filmy halo surrounding it, color blue-green. The flame on extension becomes sharply pointed at the end with little vibration. At the maximum length of four inches and blow-off pressure of three-tenths of an inch the flame became very pale.

Mixture, $\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 5 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$ Blow-off pressure, .12" H₂O.

The flame was very similar in appearance to the last with the exception of being more blue than green, the tip was very pale and sharp-pointed at the maximum length of three and one-half inches.

Mixture, $\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 6 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$ Blow-off pressure, .08" H₂O.

Flame was somewhat shorter and somewhat more filmy or cloudy in character, the maximum length of a little more than two inches was reached with a water pressure of about eight one-hundredths of an inch.

Mixtures containing more air than the last could not be burnt.

The limits of explosive combustibility differ from the preceding limits for flames. When admitted to the explosion chamber, isolated and exploded in bulk the limits of combustibility were:

$\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 3 \\ \text{Gas} \quad . \quad . \quad . \quad 1 \end{array} \right\}$ up to $\left\{ \begin{array}{l} \text{Air} \quad . \quad . \quad . \quad 7. \\ \text{Gas} \quad . \quad . \quad . \quad 1. \end{array} \right\}$

But with the mixtures 6.5/1 up to 7/1 the ignition was very uncertain, occurring only after long passage of the spark and

often failing entirely. This is an unfailing characteristic of extremely dilute mixtures, *i. e.*, mixtures containing a large percentage of neutral or excess gases.

Constant Pressure Combustion Calorimeter.

Before stating the results of this calorimeter on an unanalyzed water gas it will be well to look at some characteristics of a water of typical composition. It is intended that this calorimeter be used by men unskilled in gas analysis and in places where such an apparatus is unavailable. The results that theoretically should accrue from this typical water gas will be compressed until what was actually observed on the action of the gas used.

Stillman gives as an ordinary water gas the following mixture:

CO ₂	3.8
C ₂ H ₄	14.6
CO	28.0
H	35.6
CH ₄ ..	16.7
N	1.3
Total	100.0

of this we have

NEUTRAL.

CO ₂	3.8
N	1.3
Total	5.1

This gas, moreover, will heat yield 691.59 B. T. U. per cubic foot products condensed, and will call for in its combustion 5.21 parts of air per one part of gas.

A *chemical mixture* then would have these characteristics:

Air	5.21	volumes
Gas	1.00	"
Total	6.21	"

of which we have

$$\text{Neutral} \left\{ \begin{array}{l} \text{Neutral in gas } .051 \\ \text{Nitrogen in air } 4.120 \end{array} \right.$$

Total neutral 4.17 in 6.21 parts or 67 per cent. neutral.

Let us then tabulate various mixtures and note some of their characteristics.

Gas.	I	I	I	I	I	I	I	I	I
Air.	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
Inactive Air, <i>i. e.</i> , Excess.						.29	.79	1.29	1.79
Active Air.	3.0	3.5	4.0	4.5	5.0	5.21	5.21	5.21	5.21
Inactive Gas, <i>i. e.</i> , Excess.	.427	.328	.232	.135	.050				
Active Gas.	.573	.672	.768	.865	.950	1.000	1.000	1.000	1.000
Neutral in Active Air.	2.372	2.768	3.163	3.559	3.954	4.120	4.120	4.120	4.120
Neutral in Active Gas.	.029	.034	.039	.044	.048	.051	.051	.051	.051
Total inactive or excess.	2.828	3.130	3.434	3.735	4.052	4.461	4.961	5.461	5.961
Per cent. inactive or excess.	.701	.696	.687	.680	.675	.686	.704	.729	.746

It should be noted how very slightly the increase in percentage of dilution increases with the excesses of air and gas; though the proportions may vary over 100 per cent., the dilution varies through but little more than 5 per cent. This is very striking, as will be noted again when the results of increasing dilution by neutral additions is taken up. There can be little doubt that the limits of combustibility is intimately associated with the per cent. of neutral or inactive gases present.

Next let us examine the calorific values of some of these mixtures, *i. e.*, the amount of heat that one cubic foot of gas can deliver when burnt explosively in mixtures within the limits of explosive combustion. The heat developed by a cubic foot of the gas in question is 691.59 B. T. U. when completely burnt, *i. e.*, in a chemical mixture, or in a mixture in which air is in excess, within of course the limits of combustibility.

Gas.	I	I	I	I	I	I	I	I	I
Air.	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
Gas Burnt, <i>i. e.</i> , Gas that could find Air Enough to Burn it.	.573	.672	.768	.865	.950	1.000	1.000	1.000	1.000
B. T. U. Available.	396.3	464.8	531.2	598.2	657.0	691.6	691.6	691.6	691.6

These results are shown graphically on the curve *A* of Fig. 9. Curve *B* shows the results of observations on the water gas used in the calorimeter and of unknown composition.

The calorimeter can was filled with 16.5 lbs. water for each run and the water equivalent determined by experiment to be 2.1 lbs. of water; this is for both can and coil. Radiation was assumed zero, as the temperature of the room was in every case between the initial water temperature and the final.

Four inches altitude of gas was used every time from the eighteen-inch tank mixed each time with varying quantities of air. The results are as follows reduced to cubic feet of gas:

B. T. U. PER CUBIC FOOT OF GAS WHEN MIXED WITH AIR.

Gas.	Air.	B. T. U. per cu. ft. Gas,
I	3.0	275.1
I	3.5	347.82
I	4.0	401.57
I	4.5	471.00
I	5.0	541.70
I	5.5	616.59
I	6.0	600.78
I	6.5	?

CALORIFIC POWER of Gas-Air MIXTURES.

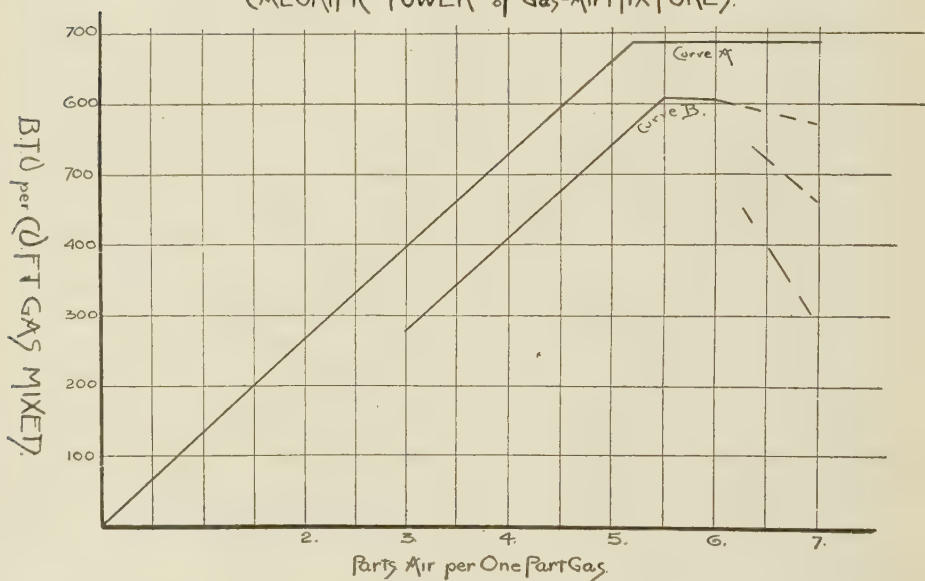


FIG. 9.

This is one set of readings, and the curve is a very good one. other sets of readings were taken, and if the results had been reduced to a mean the curve would have been perfect. The one set given were obtained in a space of about two hours. The values for mixtures 6.5/1 and 7/1 were very erratic but always below the maximum, and this result is extremely important, viz., *that very dilute mixtures after combustion has been started may cease to burn before combustion has become complete.*

It may be well to note a few peculiarities of this calorimeter before leaving the subject. When starting a continuous stream of sparks is provoked between the points and the mixture then turned on. As soon as the flame cap has settled and combustion is well started the spark may be turned off and attention turned to watching the thermometer and directing the stirring of the water. The feed may be depended upon to take care of itself. The process of starting, however, will not be successful unless good judgment in regard to certain points is exercised; however, a few trials are sufficient to show up these difficulties, and the means for avoiding them.

If the entering stream be so small in quantity as to give the gas too small a velocity through the feed pipe, then when the first part of the mixture reaches the spark there will be back-flashing, which may result in a succession of explosions. These successive explosions will have a period depending on the velocity of feed; they become more frequent as the velocity increases, until finally the velocity becomes high enough to force the flame beyond the feed pipe, when it will lodge in the rocks and stay there. It may even happen that the back-flash will extend to the trap, but no harm will be done except blowing off the rubber tube and causing a loss of the charge. At first this back-flashing was very troublesome and necessitated investigation. When the combustion chamber and coil were removed from the water no back-flash occurred even with a very slow feed, but a reinsertion caused the trouble to reappear. This was attributed to the intermittent back pressure effect of the bubbling of discharging gas through the water; it was then the flexible end to the coil was attached so that the discharge could be directed above during and below after starting, when it would do no harm. Since it was now shown that back pressure could have an appreciable effect on the action, and back-flash still occurring occasionally, the discharge gas coil of one-eighth inch copper tube shown in Fig. 1 was removed and the square coil of larger iron pipe substituted to reduce back pressure. The back pressure was thus made constant and less than originally, and the charge could be ignited at once without any failure with a constancy that was very gratifying.

Volume-ratios During Constant Pressure Combustion.

When the fire is enclosed and insulated the immediate effect of constant pressure combustion is to increase the volume of the gases (neglecting the small changes due to chemical regrouping of molecules). Knowing the amount of heat developed by the combustion and the specific heat of the gases, the volume increase should be easy to calculate. But in such a calculation no account can be taken of the large number of other influences, among them radiation, conduction, dissociation, etc., involving loss of heat to other phenomena that may be present and no assurance can be held out that the specific heat during the process is constant or equal to that of the products of combustion. A computation, therefore, by this the only method is of no value in engineering work, and the only way to obtain a result of real worth is to measure the increase directly under specified conditions. The method used has already been noted. The firebox consisted of a piece of six-inch standard steam pipe twelve inches long and the plates containing the orifices were of black iron one thirty-second of an inch in thickness. Unprotected, *i. e.*, with plates bare and pipe unlined, it was found for a pressure drop of four inches of Hg through each orifice that the maximum effect was $v_2/v_1 = 1.50$ for best mixture air and gas. Protecting the interior of the combustion chamber by one inch of fire clay and sand on the inside, the lower plate carrying a clay cone three inches high, and the upper plate protected externally by a quarter of an inch of asbestos sheets, the maximum effect was found to be $v_2/v_1 = 4.20$ for best mixture air and gas. Comparing the heating value of the gas used with that of say kerosene oil, and making an estimate of losses from above values it is *probable* that with the best mixture of kerosene and air that a value $v_2/v_1 = 6.00$ might be expected, but it must be remembered that this is only an estimate and of but little value compared with the last two figures of actual observations.

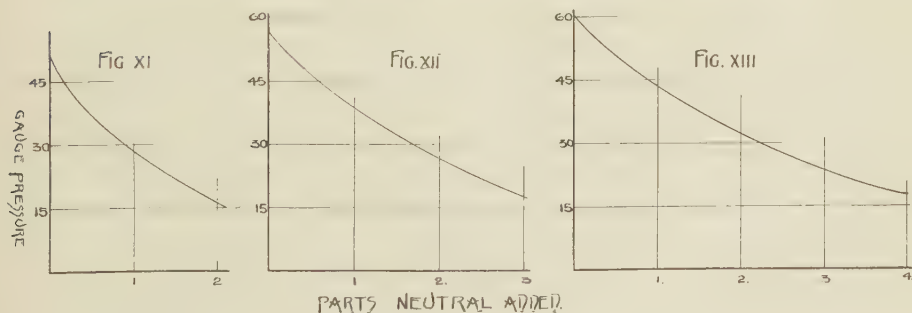
The apparatus used here was so certain in operation and constant in results that the readings could be obtained in a very short time and, when obtained, relied upon. The method of operation was as follows: The fire was started in the chamber with top plate removed; once burning steadily this plate was bolted down to

the flange and the whole allowed to heat up. The top orifice was originally the same size as the lower and as the fire pot heated up was continually reamed out to keep the manometer readings at the ratio of 2:1, *i. e.*, so that the drop in pressure was the same through both plates. When continued heating showed no increase in size of the top hole possible, then manipulation of the mixture was resorted to to cause, if possible, a rise in pressure in the combustion chamber and so permit a further enlargement of the upper opening to bring the pressure again to one half that existing in the lower chamber. Thus the maximum effect was obtained. It must be remembered that the readings included the friction effect of passing through the mass of broken rock forming the burner proper.

Pressure-Ratios for Constant Volume Combustion.

As is the case with volume ratios in constant pressure combustion it is impossible to calculate the values desired from calorific value and specific heat. Many determinations of the presence of ratios for various substances have been made but none for a wide range of mixtures including as one of the constituents neutral products of combustion.

Each mixture of air to gas within the range of combustibility was fired and then to each was added in turn successively increas-



FIGS. XI, XII, and XIII.

ing amounts of neutral gases obtained by burning an explosive mixture as described.

It appeared that the resulting pressures were intimately connected with the percentage of dilution of neutral or excess gases,

and as the gas used has already exhibited some agreement with what is possible with the typical water gas chosen in comparison it will be well to work out a table of percentage of dilution of different mixtures and these figures will be placed on the curves of Figs. 11-16. The agreement and evident existence of a law is apparent.

WATER GAS OF NOTED COMPOSITION.

Mixture, $\left\{ \begin{array}{l} \text{Air, 3} \\ \text{Gas, 1} \end{array} \right\}$ diluted.

Gas.	Air.	Added Neutral.	Primary Neutral.	Total Neutral.	Per cent. Neutral.
I	3	0	2.83	2.83	70.1
I	3	1	"	3.83	76.0
I	3	2	"	4.83	80.5
I	3	3	"	5.83	83.0

Mixture, $\left\{ \begin{array}{l} \text{Air, 4} \\ \text{Gas, 1} \end{array} \right\}$ diluted.

Gas.	Air.	Added Neutral.	Primary Neutral.	Total Neutral.	Per cent. Neutral.
I	4	0	3.43	3.43	68.7
I	4	1	"	4.43	74.0
I	4	2	"	5.43	77.6
I	4	3	"	6.43	80.6
I	4	4	"	7.43	82.8

Mixture, $\left\{ \begin{array}{l} \text{Air, 5} \\ \text{Gas, 1} \end{array} \right\}$ diluted.

Gas.	Air.	Added Neutral.	Primary Neutral.	Total Neutral.	Per cent. Neutral.
I	5	0	4.05	4.05	67.5
I	5	1	"	5.05	72.1
I	5	2	"	6.05	75.7
I	5	3	"	7.05	78.4
I	5	4	"	8.05	80.5

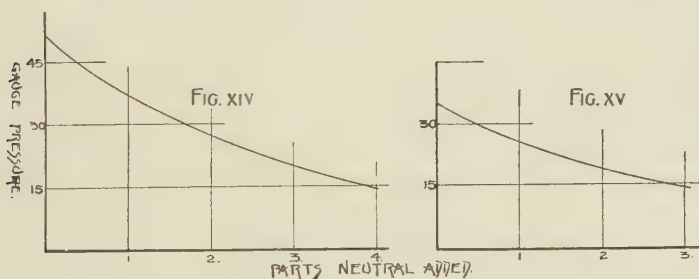
Mixture, $\left\{ \begin{array}{l} \text{Air, 6} \\ \text{Gas, 1} \end{array} \right\}$ diluted.

Air.	Gas.	Added Neutral.	Primary Neutral.	Total Neutral.	Per cent. Neutral.
I	6	0	4.96	4.96	70.4
I	6	1	"	5.96	74.4
I	6	2	"	6.96	77.3
I	6	3	"	7.96	79.6
I	6	4	"	8.96	81.3
I	6	5	"	9.96	84.0

Mixture, $\left\{ \begin{array}{l} \text{Air, 7} \\ \text{Gas, 1} \end{array} \right\}$ diluted.

Air	Gas.	Added Neutral.	Primary Neutral.	Total Neutral.	Per cent. Neutral.
I	7	0	5.96	5.96	74.6
I	7	1	"	6.96	77.3
I	7	2	"	7.96	79.6
I	7	3	"	8.96	81.3
I	7	4	"	9.96	83.0

The curves of Figs. 12-16 show the pressures given by the indicator for each mixture and are the mean values from a large number of lines drawn by the indicator. These curves are combined in Fig. 17, which is, therefore, a curve of pressures for all



FIGS. 14 and 15.

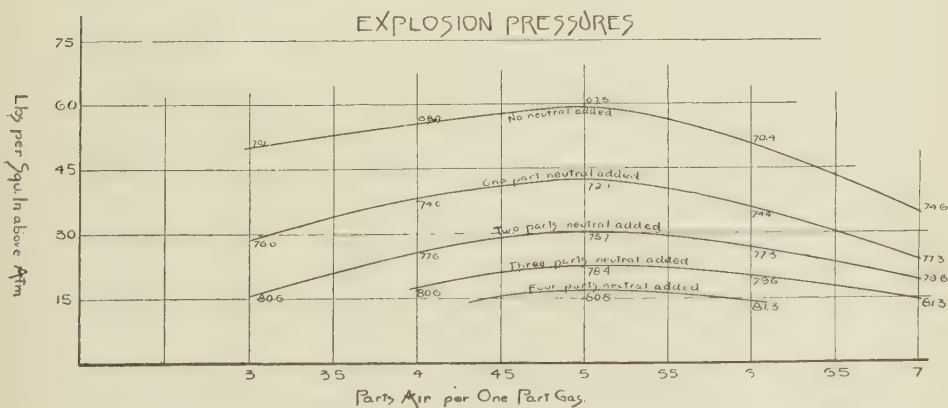


FIG. 16.

mixtures diluted or not within the range of explosive combustibility. The numbers on the curves show the percentage of dilution of the typical water gas. The results are most remarkable

and can be accounted for only by assuming that the presence of a large amount of dilution hinders combustion. The limits at which combustion ceases to be possible on too great a dilution are here indicated, whether that dilution be due to excess gas, excess air or neutral gases. It will also be observed that the character of the diluent has an appreciable effect, but that when the dilution is least the pressure is greatest, about 60 pounds above atmosphere or a ratio of 5; and the presence of a constant per cent. of neutral will make combustion impossible no matter what the mixture of air and gas. The greatest neutral dilution gives the least pressure—about 15 pounds above atmosphere, or a ratio of about 2. These results give a reason for the decreased pressure in exploding gas-engines in which the mixture is always diluted by burnt products to an extent of 20–40 per cent. of the volume of neutral addition to the gas mixture which may already have neutral gas present to the extent of 65–70 per cent.

Neutral additions to the gases sent to the calorimeter and to the other apparatus showed, besides a corresponding and proper heat value for the resulting mixture, a decreased rate of propagation accompanied by a difficulty in ignition and constant tendency to incomplete combustion, *i. e.*, tendency to cease burning after inflammation had been started and before the mass had been entirely burnt.

CONCLUSION.

The next step in this work, now that the mathematical discussion and determination of the physical action and constants is finished, is naturally to apply the results to an operating machine. It seems that the best arrangement would result in the combination of a compressor, a fire chamber and gas-expansion turbine, and it is the construction and test of such a combination that will form the subject of the next work. However, as this point marks a natural division of the subject and as sufficient has been developed to more than fill the requirements of a doctor's dissertation, the remainder of the work will be left till later, though it is with sincere regret that this investigation, already so fascinating and so fruitful, is even temporarily laid aside.

